Some Basic Concepts in Analysis of Nonlinear Dynamical Systems

Hidde de Jong

INRIA Grenoble - Rhône-Alpes, 655 Avenue de l'Europe,
Montbonnot, 38334 St. Ismier Cedex, France
http://ibis.inrialpes.fr/

INSA 5BIM

INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE | INRIA | centre de recherche GRENOBLE - RHÔNE-ALPES

Introduction

Module on regulatory networks requires knowledge of basic concepts on nonlinear dynamical systems

Aim of this course is to review these concepts by means of two-dimensional examples:

- Steady states and stability
- Limit cycles
- Bifurcations

Strogatz, *Nonlinear Dynamics and Chaos*, Addison-Wesley, 1994

Steady states and stability

Phase portrait

Phase space, vector field, trajectory, steady state, closed orbit, existence and uniqueness, ...

Stability of steady states

Stability, asymptotic stability, ...

Determination of stability

Jacobian matrix, linear system, characteristic equation, eigenvalues, classification of steady states, ...

Phase portrait

Stable and unstable manifold, nullcline, basin of attraction, separatrix, ...

Lotke-Volterra model of competition

Classical model used to describe competition between two species in population dynamics

Example: two species (*e.g.*, rabbits and sheep) that are competing for the same food supply (*e.g.*, grass)

Lotke-Volterra model of competition

$$\dot{x} = r_1 x(1 - \frac{x}{k_1} - b_1 \frac{y}{k_1})$$
$$\dot{y} = r_2 y(1 - \frac{y}{k_2} - b_2 \frac{x}{k_2})$$

- $x, y \geq 0$: population size
- $r_1, r_2 \geq 0$: maximum growth rates
- $k_1, k_2 \geq 0$: carrying capacities
- $b_1, b_2 \geq 0$: competition parameters

Edelstein-Keshet, *Mathematical Models in Biology*, SIAM, 2005

Limit cycles and bifurcations

Limit cycles

Stable and unstable limit cycles, ...

Finding limit cycles

Poincaré-Bendixson theorem, trapping region, ...

Glycolytic oscillations

Glycolysis is fundamental biochemical process concerned with breakdown of sugar to yield energy and precursors of macromolecules

In some species, under specific conditions, oscillations have been shown to occur

Goldbeter, *Biochemical Oscillations and Cellular Rhythms*, Cambridge University Press, 1997

Minimal model of glycolytic oscillations

$$\dot{x} = -x + ay + x^2 y$$
$$\dot{y} = b - ay - x^2 y$$

- $x, y \geq 0$: concentrations of adenosine diphosphate (ADP) and fructose-6-phosphate (F6P)
- $a, b \geq 0$: kinetic parameters

Sel'kov, *Eur. J. Biochem.*, 4:79-86, 1968

Bifurcations

Bifurcations

Bifurcation, different types of bifurcation, ...

Auto-activation in gene regulation

Proteins may activate expression of their own gene

Ubiquitous motif in gene regulatory networks, see later courses

Classical model of auto-activation in gene expression

$$\dot{x} = -ax + y$$

$$\dot{y} = \frac{x^2}{1 + x^2} - by$$

- $x, y \geq 0$: concentrations of protein and mRNA
- $a, b \geq 0$: degradation parameters

Griffith, *Mathematical Neurobiology*, Academic Press, 1971