Qualitative Modeling and Simulation of Genetic Regulatory Networks

Hidde de Jong



INRIA Grenoble - Rhône-Alpes

Hidde.de-Jong@inria.fr



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- IBIS: systems biology group of INRIA and Joseph Fourier University/CNRS
 - Analysis of bacterial regulatory networks by means of models and experiments
 - Involves molecular biologists, mathematicians, physicists, computer scientists, ...







- 1. Genetic regulatory networks in bacteria
- 2. Qualitative simulation of genetic regulatory networks using piecewise-linear models
- 3. Qualitative simulation of carbon starvation response in *Escherichia coli*: model predictions and validation
- 4. Perspectives: towards quantitative models





Bacterial growth and adaptation

Bacteria are geared towards growth and division

E. coli cells have doubling times up to 20 min



Stewart et al. (2005), PLoS Biol., 3(2): e45

Changes in environment may cause adaptation of growth rate, and more generally, physiology of bacterial cell

Nutrient starvation, heat shock, osmotic stress, high population density, ...





Molecular basis of growth adaptation

Adaptation of growth rate and other stress responses involves complex regulatory networks

Network of biochemical reactions underlying carbon assimilation in E. coli



Baldazzi et al. (2010), PLoS Comput. Biol., 6(6):e1000812



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Different time-scales, different networks

- Variety of biochemical reactions, occurring on different timescales, define different types of networks
 - Signal transduction networks (ms), metabolic networks (s), gene regulatory networks (min)





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Gene regulatory networks

Gene regulatory networks control long-term adaptation of bacterial cell to external perturbations

Genes, gene products (RNAs, proteins), and the regulatory effect of the

latter on the expression of other genes

Bolouri (2008), *Computational Modeling of Gene Regulatory Networks*, Imperial College Press

Gene regulatory networks cannot be reduced to direct interactions (transcription regulation), but also include indirect interactions (mediated by metabolism)

Brazhnik et al. (2002), Trends Biotechnol., 20(11):467-72



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Structure of gene regulatory networks



Structure of gene regulatory networks can be obtained by reduction of kinetic model of system of biochemical reactions

Distinct time-scale hierarchies between gene expression and metabolism Baldazzi *et al.* (2010), *PLoS Comput. Biol.*, 6(6):e1000812





Structure of gene regulatory networks



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Dynamics of gene regulatory networks



No global view of functioning of most gene regulatory network available, despite abundant knowledge on network components

Understanding of dynamics requires **mathematical modeling** and **computer analysis and simulation**





Hierarchy of modeling formalisms

Variety of modeling formalisms exist, describing system on different levels of detail

> de Jong (2002), *J. Comput. Biol.*, 9(1): 69-105 Hasty *et al.* (2001), *Nat. Rev. Genet.*, 2(4):268-279 Smolen *et al.* (2000), *Bull. Math. Biol.*, 62(2):247-292 Szallassi et al. (2006), *System Modeling in Cellular Biology*, MIT Press







Modeling of genetic regulatory network

Well-established theory for modeling of genetic regulatory networks using ordinary differential equation (ODE) models

> Bolouri (2008), *Computational Modeling of Gene Regulatory Networks*, Imperial College Press Polynikis *et al.* (2009), *J. Theor. Biol.*, 261(4):511-30

Practical problems encountered by modelers:

- Knowledge on molecular mechanisms rare
- Quantitative information on kinetic parameters and molecular concentrations absent
- Large models
- Even in the case of well-studied E. coli network!



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Qualitative modeling and simulation

- Possible strategies to overcome problems
 - Parameter estimation from experimental data
 - Parameter sensitivity analysis
 - Model simplifications
- Intuition: essential properties of network dynamics robust against reasonable model simplifications
- Qualitative modeling and simulation of large and complex genetic regulatory networks using simplified models de Jong, Gouzé et al. (2004), Bull. Math. Biol., 66(2):301-40
- Relation with discrete, logical models of gene regulation

Thomas and d'Ari (1990), *Biological Feedback*, CRC Press Kauffman (1993), *The Origins of Order*, Oxford University Press





Ordinary differential equation models

Genetic regulatory networks modeled by ODE models using sigmoid functions to describe regulatory interactions

$$\dot{x}_a = \kappa_a h^{-}(x_a, \theta_{a2}, n) h^{-}(x_b, \theta_b, n) - \gamma_a x_a$$
$$\dot{x}_b = \kappa_b h^{-}(x_a, \theta_{a1}, n) - \gamma_b x_b$$





- x: protein concentration
- θ : threshold concentration
- κ , γ : rate constants
- *n* : steepness parameter
- Expressions of sigmoid functions account for combinatorial control of gene expression (AND, OR, NOR, ...)





PL differential equation models

ODE models approximated by means of step functions to describe regulatory interactions

$$\dot{x}_a = \kappa_a \ s^{-}(x_a, \ \theta_{a2}) \ s^{-}(x_b, \ \theta_b) - \gamma_a \ x_a$$
$$\dot{x}_b = \kappa_b \ s^{-}(x_a, \ \theta_{a1}) - \gamma_b \ x_b$$





- x: protein concentration
- θ : threshold concentration
- κ , γ : rate constants
- Piecewise-linear (PL) DE models of genetic regulatory
 networks
 Glass and Kauffman (1973), J. Theor. Biol., 39(1):103-29





Analysis of local dynamics of PL models

Monotone convergence towards **focal point** in regions separated by thresholds



Glass and Kauffman (1973), J. Theor. Biol., 39(1):103-29



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Analysis of local dynamics of PL models

Monotone convergence towards **focal point** in regions separated by thresholds



Glass and Kauffman (1973), J. Theor. Biol., 39(1):103-29





Analysis of local dynamics of PL models

Instantaneous crossing of regions located on thresholds, or ...





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- Analysis of local dynamics of PL models
 - ... quasi-monotone convergence towards focal sets located on threshold

hyperplanes max_{b} θ_{b} θ_{b} θ_{al} θ_{al} θ_{a2} max_{a} $\dot{x}_{a} = \kappa_{a} s^{-}(x_{a}, \theta_{a2}) s^{-}(x_{b}, \theta_{b}) - \gamma_{a} x_{a}$ $\dot{x}_{b} = \kappa_{b} s^{-}(x_{a}, \theta_{a1}) - \gamma_{b} x_{b}$

Extension of PL differential equations to differential inclusions
 using Filippov approach
 Gouzé and Sari (2002), Dyn. Syst., 17(4):299-316



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- Analysis of global dynamics obtained by piecing together local dynamics in regions
 - PL approximation preserves bistability of cross-inhibition network





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- State space can be partitioned into regions with unique derivative sign pattern
- Qualitative abstraction yields state transition graph that provides discrete picture of continuous dynamics

Alur et al. (2000), Proc. IEEE, 88(7):971-84



de Jong *et al.* (2004), *Bull. Math. Biol.*, 66(2):301-40 Batt *et al.* (2008), *Automatica*, 44(4):982-9





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- State transition graph gives conservative approximation of continuous dynamics
 - Every solution of PL model corresponds to path in state transition graph
 - Converse is not necessarily true!
- State transition graph is invariant for given inequality constraints on parameters



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 - Converse is not necessarily true!
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Use of state transition graph

- Analysis of steady states and limit cycles of PL models
 - Attractor states in graph correspond (under certain conditions) to stable
 steady states of PL model
 Casey et al. (2006), J. Math Biol., 52(1):27-56
 - Attractor cycles in graph correspond (under certain conditions) to stable
 limit cycles of PL model
 Glass and Pasternack (1978), J. Math Biol., 6(2):207-23

Edwards (2000), *Physica D*, 146(1-4):165-99





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Use of state transition graph

- Paths in state transition graph represent predicted sequences of qualitative events
- Model validation: comparison of predicted and observed sequences of qualitative events



Need for automated and efficient tools for model validation





Model validation by model checking

Dynamic properties of system can be expressed in temporal logic (CTL)

There *E*xists a *F*uture state where $\dot{x}_a > 0$ and $\dot{x}_b > 0$ and starting from that state, there *E*xists a *F*uture state where $\dot{x}_a < 0$ and $\dot{x}_b > 0$

$$\boldsymbol{EF}(\dot{\boldsymbol{x}}_a > 0 \land \dot{\boldsymbol{x}}_b > 0 \land \boldsymbol{EF}(\dot{\boldsymbol{x}}_a < 0 \land \dot{\boldsymbol{x}}_b > 0))$$



Model checking is automated technique for verifying that state transition graph satisfies temporal-logic statements

Efficient computer tools available for model checking

Batt et al. (2005), Bioinformatics, 21(supp. 1): i19-i28





Genetic Network Analyzer (GNA)

Qualitative analysis of PL models implemented in Java: Genetic Network Analyzer (GNA)



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Distribution by Genostar SA



de Jong *et al.* (2003), *Bioinformatics*, 19(3):336-44

http://www-helix.inrialpes.fr/gna

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Genetic Network Analyzer (GNA)

- Model-checking technology made available to GNA user
 - Develop temporal logics tailored to biological questions

Mateescu *et al.* (2010), *Theor. Comput. Sci., in press*

 Develop temporal-logic patterns patterns for frequently-asked modeling questions



Monteiro et al. (2008), Bioinformatics, 24(16):i227-33

Connect GNA to standard model checkers through a web-server
 Connection
 Monteiro *et al.*, (2009), *BMC Bioinform.*, 10:450





Analysis of bacterial regulatory networks

- Applications of qualitative simulation in bacteria:
 - Initiation of sporulation in *Bacillus subtilis* de Jong, Geiselmann *et al.* (2004), *Bull. Math. Biol.*, 66(2):261-300
 - Quorum sensing in *Pseudomonas* aeruginosa

Viretta and Fussenegger (2004), *Biotechnol. Prog.*, 20(3):670-8

Onset of virulence in *Erwinia chrysanthemi*

Sepulchre et al. (2007), J. Theor. Biol., 244(2):239-57







Echerichia coli

Enteric bacterium *Escherichia coli* has been most-studied organism in biology

« All cell biologists have two cells of interest: the one they are studying and Escherichia coli »

Schaechter and Neidhardt (1996), in: Escherichia coli and Salmonella, ASM Press, 4



Stress responses in Escherichia coli

- Enterobacterium *E. coli* is able to adapt to a variety of stresses in its environment
 - Model organism for understanding of decision-making processes in single-cell organisms
 - Model organism for understanding adaptation of pathogenic bacteria to their host
 Storz and Hengge-Aropis (2000), Bacterial Stress Responses, ASM Press





E. coli response to carbon starvation

Response of *E. coli* to carbon starvation conditions: transition from exponential phase to stationary phase



 Growth transition is accompanied by profound changes in gene expression, allowing cell to adjust its functioning to stress
 Storz and Hengge-Aronis (2000), Bacterial Stress Responses, ASM Press



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Modeling of carbon starvation network

Can we understand how gene regulatory network controls adaptation in response to carbon starvation?

Network senses carbon source availability and **global regulators** coordinate adaptive response of bacteria





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Modeling of carbon starvation network

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Development of qualitative model of network

Translation of network diagram into PL formalism (regulatory logic)

Ropers et al. (2006), Biosystems, 84(2):124-152; Ropers et al. (2010), in press





Development of PL model

- Translation of network diagrams into PL models
 - Straightforward for direct interactions...





• ... but also possible for indirect interactions







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PL model of carbon starvation network

$$\begin{split} \dot{u}_{s} &= 0 \\ \dot{x}_{y} &= \kappa_{y}^{1} + \kappa_{y}^{2} \left(1 - s^{+}(x_{c},\theta_{c}^{2}) s^{+}(x_{y},\theta_{y}^{2}) s^{+}(u_{s},\theta_{s}) \right) - \gamma_{y} x_{y} \\ \dot{x}_{c} &= \kappa_{c}^{1} + \kappa_{c}^{2} s^{-}(x_{f},\theta_{f}^{2}) s^{+}(x_{c},\theta_{c}^{1}) s^{+}(x_{y},\theta_{y}^{1}) s^{+}(u_{s},\theta_{s}) + \kappa_{c}^{3} s^{-}(x_{f},\theta_{f}^{1}) - \gamma_{c} x_{c} \\ \dot{x}_{f} &= \kappa_{f}^{1} \left(1 - s^{+}(x_{c},\theta_{c}^{1}) s^{+}(x_{y},\theta_{y}^{1}) s^{+}(u_{s},\theta_{s}) \right) s^{-}(x_{f},\theta_{f}^{6}) \\ &+ \kappa_{f}^{2} s^{+}(x_{a},\theta_{a}^{1}) s^{-}(x_{i},\theta_{i}^{1}) s^{-}(x_{t},\theta_{f}^{1}) \\ \times \left(1 - s^{+}(x_{c},\theta_{c}^{1}) s^{+}(x_{y},\theta_{y}^{1}) s^{+}(u_{s},\theta_{s}) \right) - \gamma_{f} x_{f} \\ \dot{x}_{a} &= \kappa_{a} \left(1 - s^{+}(x_{a},\theta_{a}^{2}) s^{-}(x_{i},\theta_{i}^{2}) s^{-}(x_{f},\theta_{f}^{4}) - \gamma_{a} x_{a} \\ \dot{x}_{i} &= \kappa_{i} s^{+}(x_{c},\theta_{c}^{1}) s^{+}(x_{y},\theta_{y}^{1}) s^{+}(u_{s},\theta_{s}) s^{+}(x_{o},\theta_{o}) - \gamma_{i} x_{i} \\ \dot{x}_{o} &= \kappa_{o} - \left(\gamma_{o} + k_{7} s^{+}(x_{b},\theta_{b}) s^{-}(u_{s},\theta_{s}) \right) x_{o} \\ \dot{x}_{b} &= \kappa_{b}^{1} + \kappa_{b}^{2} s^{+}(x_{o},\theta_{o}) - \gamma_{b} x_{b} \\ \dot{x}_{t} &= \kappa_{t}^{1} s^{+}(x_{a},\theta_{a}^{3}) s^{-}(x_{i},\theta_{i}^{3}) s^{+}(x_{f},\theta_{f}^{5}) + \kappa_{t}^{2} s^{+}(x_{o},\theta_{o}) - \gamma_{t} x_{t} \\ \dot{x}_{n} &= \kappa_{n}^{1} s^{+}(x_{f},\theta_{f}^{3}) + \kappa_{n}^{2} - \gamma_{n} x_{n} \end{split}$$

PL models supplemented with inequality constraints on parameter values

Inequality constraints inferred from experimental literature



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Attractors of stress response network

- Analysis of attractors of PA model: two steady states
 - Stable steady state, corresponding to exponential-phase conditions



Transition to stationary phase

Does model reproduce transition from exponential phase to stationary phase upon carbon starvation?





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Qualitative simulation of network

Simulation of transition from exponential to stationary phase State transition graph with 851 states starting from exponential phase, all paths converge to stationary-phase steady state upon stress signal



Insight into carbon starvation response

Sequence of qualitative events leading to adjustment of growth of cell after carbon starvation signal



Central role for mutual inhibition of Fis and Crp, the two major regulators of the cell

Cross inhibition functions as toggle switch for adjusting expression level of genes controlled by Fis and Crp, switch pulled by stress signal





Reporter gene systems

- Simulations yield predictions that cannot be verified with currently available experimental data
- Use of reporter gene systems to monitor gene expression





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Real-time monitoring of gene expression

Integration of reporter gene systems into bacterial cell







Real-time monitoring of gene expression

Integration of reporter gene systems into bacterial cell



96-well microplate

Use of automated microplate reader to monitor in parallel in single experiment expression of different reporter genes





Reporter gene measurements

- High-precision measurements of changes in gene expression on population level in response to environmental perturbations
 - Measurement of absorbance, fluorescence, luminescence
 - About 100 data points over an interval of 10 hours
- Treatment of raw data
 - Outlier detection, regression spline fitting with GCV, background substraction, confidence intervals via bootstrap, ...

Fluorescence and absorbance data





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de Jong et al. (2010), BMC Syst. Biol., 4:55

Reporter gene measurements

Computation of biological quantities using kinetic models Reporter protein concentrations and reporter synthesis rates (proportional to mRNA concentrations)



$$\frac{dn(t)}{dt} = \kappa_m f(t) - (\mu(t) + \gamma_n) n(t)$$
$$\frac{dq(t)}{dt} = \kappa_p n(t) - (\mu(t) + \gamma_q) q(t)$$
$$\frac{dr(t)}{dt} = \kappa_r (q(t) - r(t)) - (\mu(t) + \gamma_q) r(t)$$



Validation of reporter gene measurements

Good correspondence between (relative) protein concentrations and protein synthesis rates (mRNA concentrations) and direct measurements



de Jong et al. (2010), BMC Syst. Biol., 4:55

Corrections for differences in degradation constants slightly improves correspondence



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Analysis of reporter gene expression data

 Wellreader: Matlab program for analysis of reporter gene expression data







Qualitative models can help understand basic principles underlying dynamics of complex gene regulatory networks

Models provide a coarse-grained picture of the dynamics, but capture the regulatory logic of the network of interactions

- Biological validation of qualitative models has a long history, and experimental validation of specific predictions is increasingly becoming possible
 - New measurement techniques: fluorescent reporter genes, plate readers (cell populations) and microscopes (individual cells), quantitative proteomics and metabolomics, ...
 - New ways to **control biological systems**: overexpression plasmids, microfluidics, synthetic biology, ...





- Quality of data produced by new experimental techniques makes it increasingly realistic to work with fully quantitative ODE models
 - More accurate representation of biochemical mechanisms
 - More precise predictions
- Perspective raises new methodological challenges: parameter estimation in large nonlinear ODE models





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Coupling of global regulation of transcription with carbon metabolism

Modeling of glucose-acetate shift in E. coli





Kotte et al. (2010), Mol. Syst. Biol., 6: 355

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Kinetic model with 47 variables and 193 parameters

Parameters estimated from published experimental steady-state data sets for balanced growth on either glucose or acetate

Analysis of model shows that adaptation to change in carbon source is achieved by distributed sensing of intracellular fluxes





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Kotte et al. (2010), Mol. Syst. Biol., 6: 355

No algorithms that guarantee globally optimal solution for parameter estimation in nonlinear models

Evolutionary algorithms, simulated annealing, genetic algorithms, ...

Parameter estimation demands experimental data of sufficient quantity and quality

Common problems: noise, sampling density, unobserved variables, ...

Ashyraliyev et al. (2009), *FEBS J.*, 276:886-902 van Riel (2006), *Brief. Bioinform.*, 7(4):364–74

Moreover, models of regulatory networks may be nonidentifiable by principle, but …

... even partially identifiable models may yield interesting results!



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Conclusions

- Modeling of genetic regulatory networks in bacteria often hampered by lack of information on parameter values
- Use of coarse-grained PL models that provide reasonable approximation of dynamics
- Mathematical methods and computer tools for analysis of qualitative dynamics of PL models

Weak information on parameter values (inequality constraints)

- Use of PL models may gain insight into functioning of large and complex networks
- PL models provide first idea of qualitative dynamics that may guide modeling by means of quantitative models





Contributors and sponsors

Grégory Batt, INRIA Paris-Rocquencourt Valentina Baldazzi, INRA Avignon Bruno Besson, INRIA Grenoble-Rhône-Alpes Eugenio Cinquemani, INRIA Grenoble-Rhône-Alpes Hidde de Jong, INRIA Grenoble-Rhône-Alpes Estelle Dumas, INRIA Grenoble-Rhône-Alpes Johannes Geiselmann, Université Joseph Fourier, Grenoble Jean-Luc Gouzé, INRIA Sophia-Antipolis-Méditerranée Radu Mateescu, INRIA Grenoble-Rhône-Alpes Pedro Monteiro, INRIA Grenoble-Rhône-Alpes/IST Lisbon Michel Page, INRIA Grenoble-Rhône-Alpes/Université Pierre Mendès France, Grenoble Corinne Pinel, Université Joseph Fourier, Grenoble Caroline Ranquet, Université Joseph Fourier, Grenoble Delphine Ropers, INRIA Grenoble-Rhône-Alpes









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Courtesy Guillaume Baptist (2008)



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