Some Basic Concepts in Analysis of Nonlinear Dynamical Systems

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INRIA Grenoble - Rhône-Alpes and IBIS

- IBIS: systems biology group at INRIA/Université Joseph Fourier/CNRS
  - Analysis of bacterial regulatory networks by means of models and experiments
  - Biologists, computer scientists, mathematicians, physicists, …

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Introduction

- Module on regulatory networks requires knowledge of basic concepts on nonlinear dynamical systems
- Aim of this course is to review these concepts by means of two-dimensional examples:
  - Steady states and stability
  - Limit cycles
  - Bifurcations

Strogatz, *Nonlinear Dynamics and Chaos*, Addison-Wesley, 1994

Steady states and stability

- Phase portrait

  Phase space, vector field, trajectory, steady state, closed orbit, existence and uniqueness, ...

- Stability of steady states

  Stability, asymptotic stability, ...

- Determination of stability

  Jacobian matrix, linear system, characteristic equation, eigenvalues, classication of steady states, ...

- Phase portrait

  Stable and unstable manifold, nullcline, basin of attraction, separatrix, ...

Lotka-Volterra model of competition

- Classical model used to describe competition between two species in population dynamics
  - Example: two species (e.g., rabbits and sheep) that are competing for the same food supply (e.g., grass)
- Lotka-Volterra model of competition

$$\dot{x} = r_1 x \left(1 - \frac{x}{k_1} - b_1 \frac{y}{k_1}\right)$$
$$\dot{y} = r_2 y \left(1 - \frac{y}{k_2} - b_2 \frac{x}{k_2}\right)$$

  - $x, y \geq 0$: population sizes
  - $r_1, r_2 \geq 0$: maximum growth rates
  - $k_1, k_2 \geq 0$: carrying capacities
  - $b_1, b_2 \geq 0$: competition parameters

Edelstein-Keshet, *Mathematical Models in Biology*, SIAM, 2005

Limit cycles and bifurcations

- Limit cycles

  Stable and unstable limit cycles, ...

- Finding limit cycles

  Poincare-Bendixson theorem, trapping region, ...

Glycolytic oscillations

- Glycolysis is fundamental biochemical process concerned with breakdown of carbon sources to yield energy and precursors of macromolecules
- In some species, under specic conditions, oscillations have been shown to occur

Goldbeter, *Biochemical Oscillations and Cellular Rhythms*, Cambridge University Press, 1997

- Minimal model of glycolytic oscillations

$$\dot{x} = -x + ay + x^2 y$$

$$\dot{y} = b - ay - x^2 y$$

  - $x, y \geq 0$: concentrations of adenosine diphosphate (ADP) and fructose-6-phosphate (F6P)
  - $a, b \geq 0$: kinetic parameters

Sel'kov, *Eur. J. Biochem.*, 4:79-86, 1968

Bifurcations

- Bifurcations

  Bifurcation, different types of bifurcation, ...

Auto-activation in gene regulation

- Proteins may activate expression of their own gene
  - Ubiquitous motif in gene regulatory networks, see later courses
- Classical model of auto-activation in gene expression

$$\dot{x} = -ax + y$$
$$\dot{y} = \frac{x^2}{1+x^2} - by$$

– $x, y \geq 0$: concentrations of protein and mRNA
– $a, b \geq 0$: degradation parameters

Griffith, *Mathematical Neurobiology*, Academic Press, 1971

Merci

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