



# Modeling and simulation of gene regulatory networks 5

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# INRIA Grenoble - Rhône-Alpes and IBIS



- IBIS: systems biology group at INRIA/Université Joseph Fourier/CNRS
  - Analysis of bacterial regulatory networks by means of models and experiments
  - Biologists, computer scientists, mathematicians, physicists, ...

<http://ibis.inrialpes.fr>



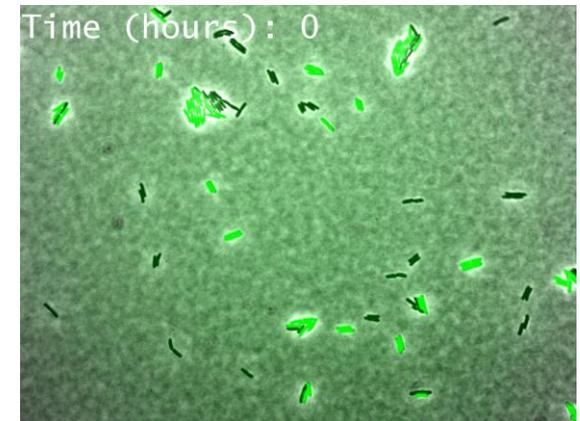
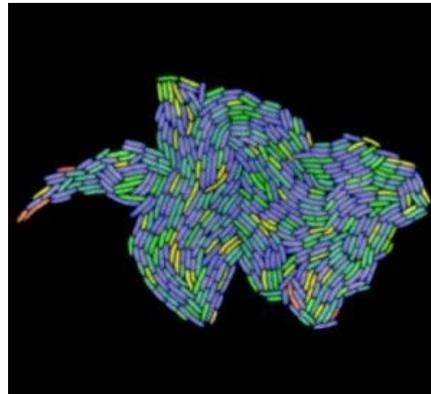
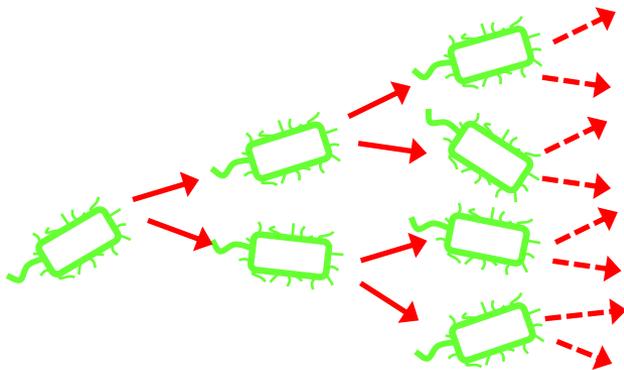
# Overview

1. Gene regulatory networks in bacteria
2. Deterministic modeling of gene regulatory networks
3. Qualitative modeling of gene regulatory networks
4. Stochastic modeling of gene regulatory networks
5. **Some current issues and perspectives**
  - Global physiological effects on the dynamics of gene expression
  - Strategies for dealing with incomplete information: the case of *Drosophila* development

# Bacterial growth and metabolism

- Bacteria are geared towards **growth** and **division**

*Escherichia coli* cells have doubling times up to 20 min



G. Baptist

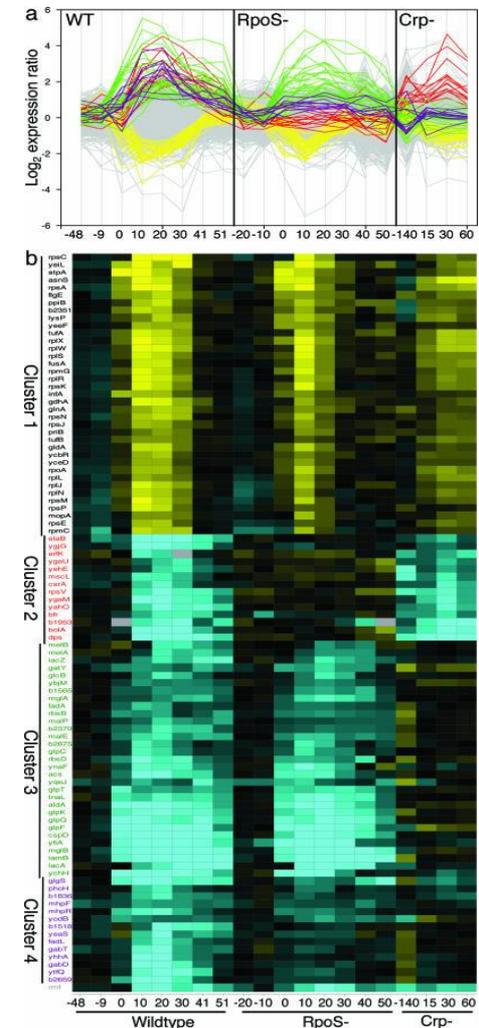
- **Metabolism** fuels growth by production of energy and building blocks for macromolecules, using nutrients from environment

ATP, amino acids, nucleotides, ...

# Growth transition and gene expression

- Genome-wide reorganization of gene expression following growth transitions in bacteria

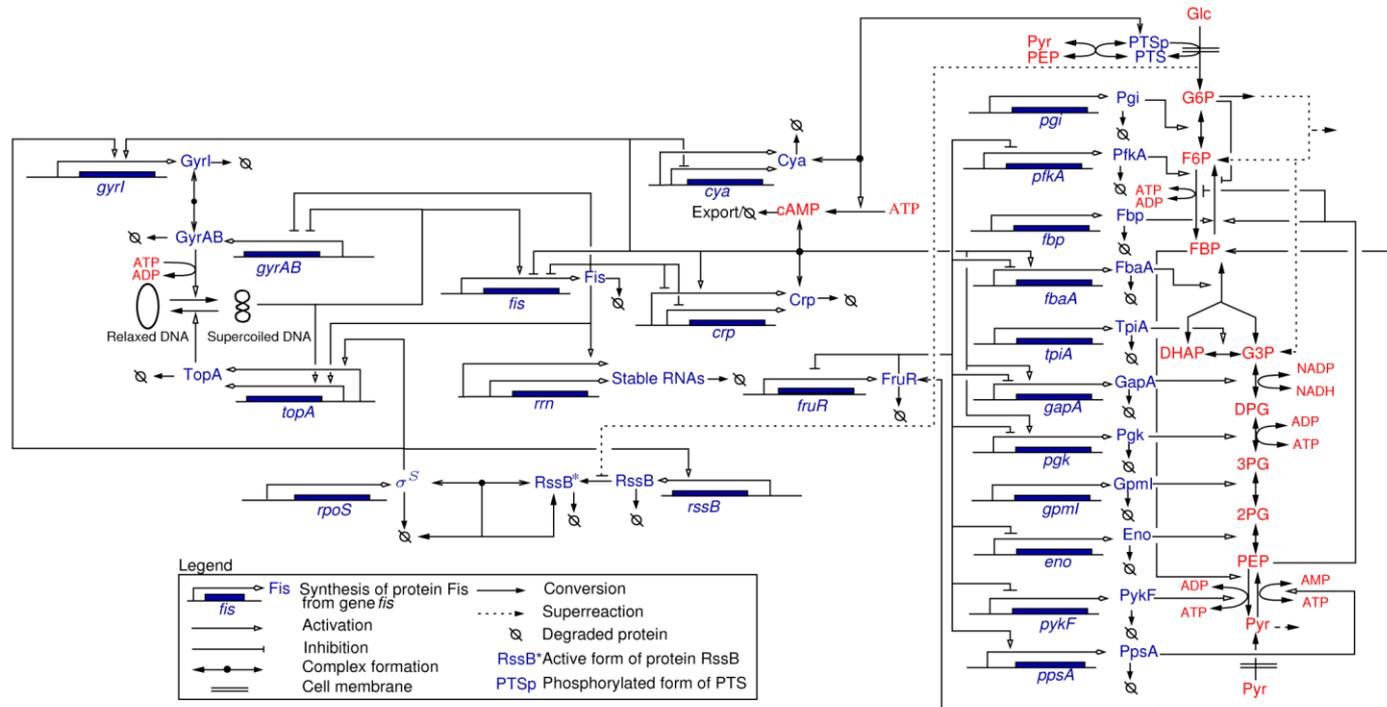
Gene expression during glucose-lactose diauxie in *E. coli*



Traxler *et al.* (2006), *Proc. Natl. Acad. Sci. USA*, 103(7):2374–9

# Growth transition and gene expression

- Adjustment of gene expression involves variety of specific regulators
  - Transcription factors, small regulatory RNAs, ...
- Complex regulatory networks control adaptive responses of cell

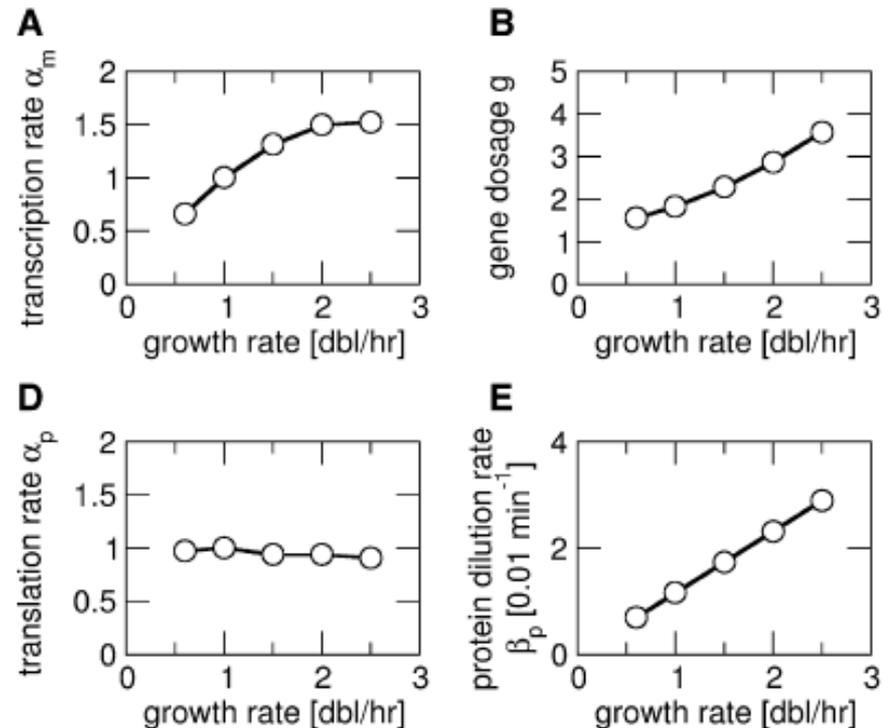


Baldazzi et al. (2010), *PLoS Comput. Biol.*, 6(6):e1000812

# Growth transition and gene expression

- Adjustment of gene expression also involves global physiological effects

Abundance of transcriptional and translational machinery, size of metabolic pools, gene copy number, ...



Klumpp *et al.* (2009), *Cell*, 139(7):1366-75

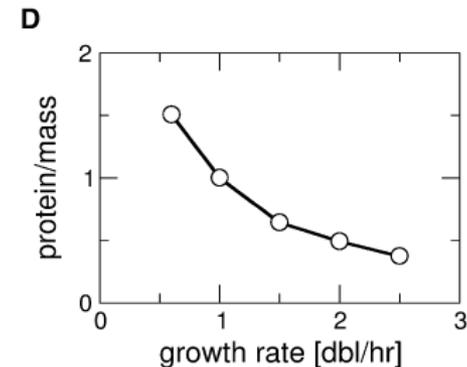
Bremer and Dennis (1996), *Escherichia Coli and Salmonella*, ASM Press, 1553-69

# Growth transition and gene expression

- **Question:** what are relative contributions of specific regulators and global physiological effects in adaptation of gene expression during growth transitions?

# Growth transition and gene expression

- **Question:** what are relative contributions of specific regulators and global physiological effects in adaptation of gene expression during growth transitions?
- Previous work on growth-rate dependent expression of **constitutive** and regulated genes
  - Constitutive gene: expression is controlled by global physiology, but not by specific transcription factors
  - Expression of constitutive gene is growth-rate dependent



Klumpp *et al.* (2009), *Cell*, 139(7):1366-75



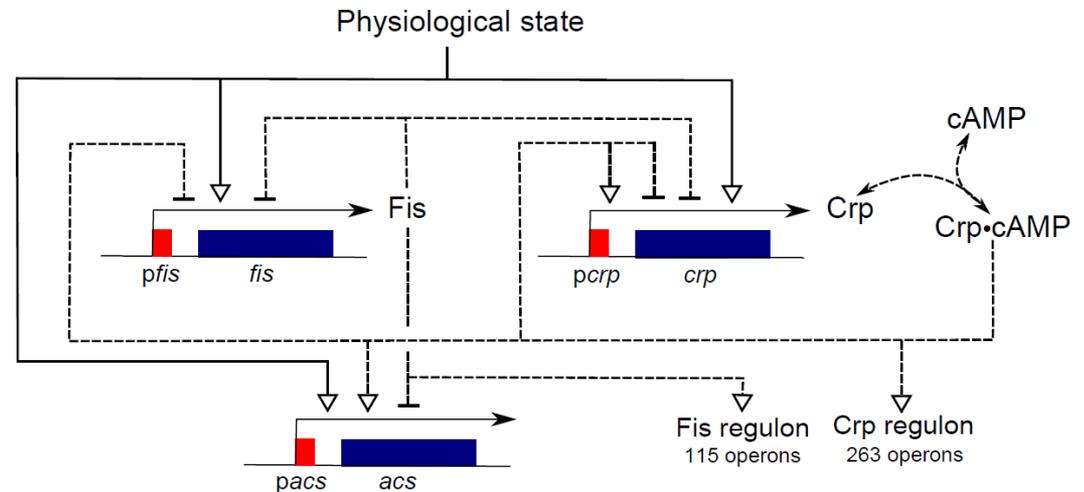
# Growth transition and gene expression

- **Question:** what are relative contributions of specific regulators and global physiological effects in adaptation of gene expression during growth transitions?  
**Dynamics** instead of steady-state, **network** instead of single gene

Berthoumieux *et al.* (2013), *Mol. Syst. Biol.*, 9:634

# Growth transition and gene expression

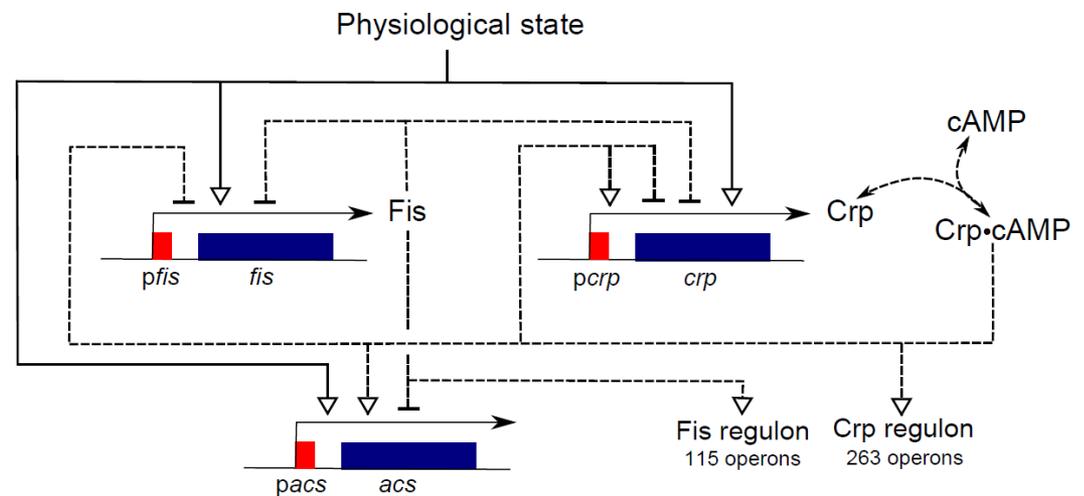
- **Question:** what are relative contributions of specific regulators and global physiological effects in adaptation of gene expression during growth transitions?
  - **Dynamics** instead of steady-state, **network** instead of single gene
- Question addressed in context of central regulatory circuit of carbon metabolism in *E. coli*



Berthoumieux *et al.* (2013), *Mol. Syst. Biol.*, 9:634

# Approach

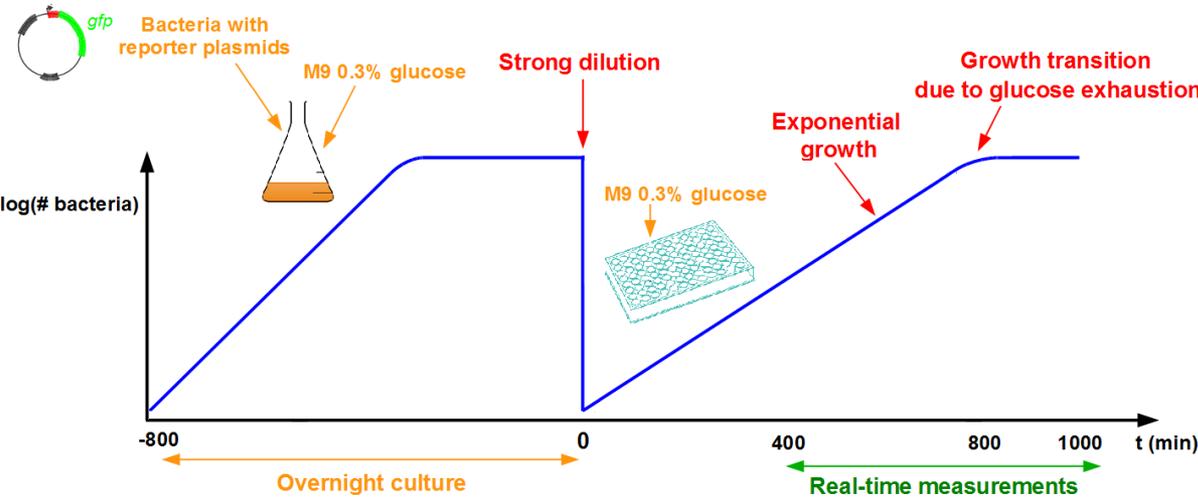
- **Real-time monitoring of dynamic response** of network to depletion of carbon source (glucose):
  - Growth rate
  - cAMP concentration
  - Promoter activity of network genes
  - Global physiological state through use of constitutive phage promoter



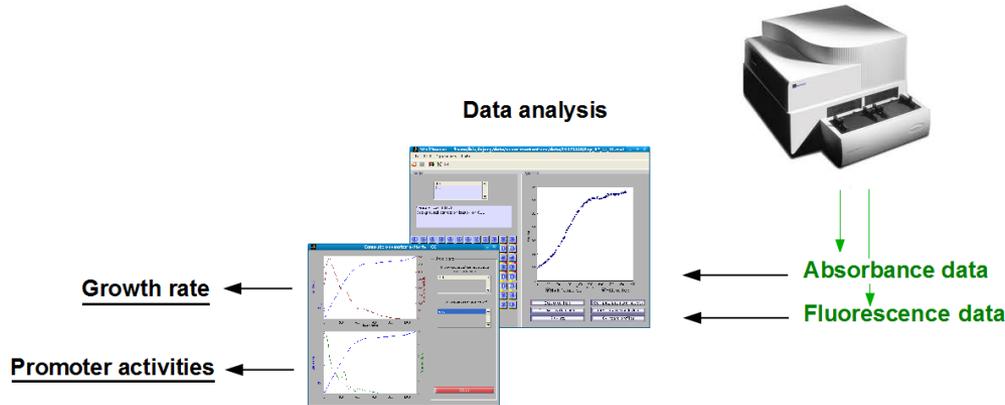
# Approach

- **Real-time monitoring of dynamic response** of network to depletion of carbon source (glucose):
  - Growth rate
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- **Simple models of promoter activities** of network genes
  - Models represent different hypotheses on contributions from global and specific effects
- **Validation of models** using experimental data

# Real-time monitoring of gene expression



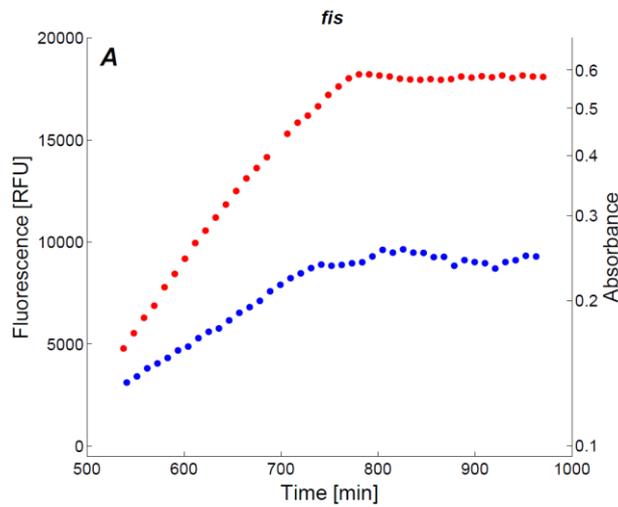
- Transcriptional fusion of promoters with *gfp* reporter genes on plasmid
- Measurement of absorbance and fluorescence signals, thermostated automated microplate reader
- Model-based derivation of promoter activities



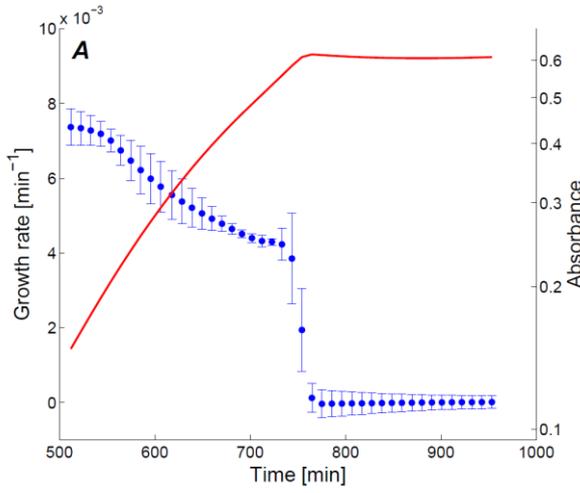
de Jong *et al.* (2010), *BMC Syst. Biol.*, 4:55

# Real-time monitoring of gene expression

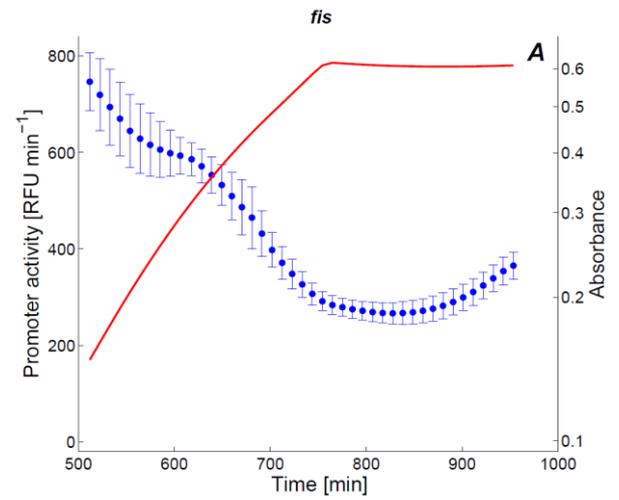
- Monitoring of *fis* promoter activity during growth transition



Absorbance and fluorescence



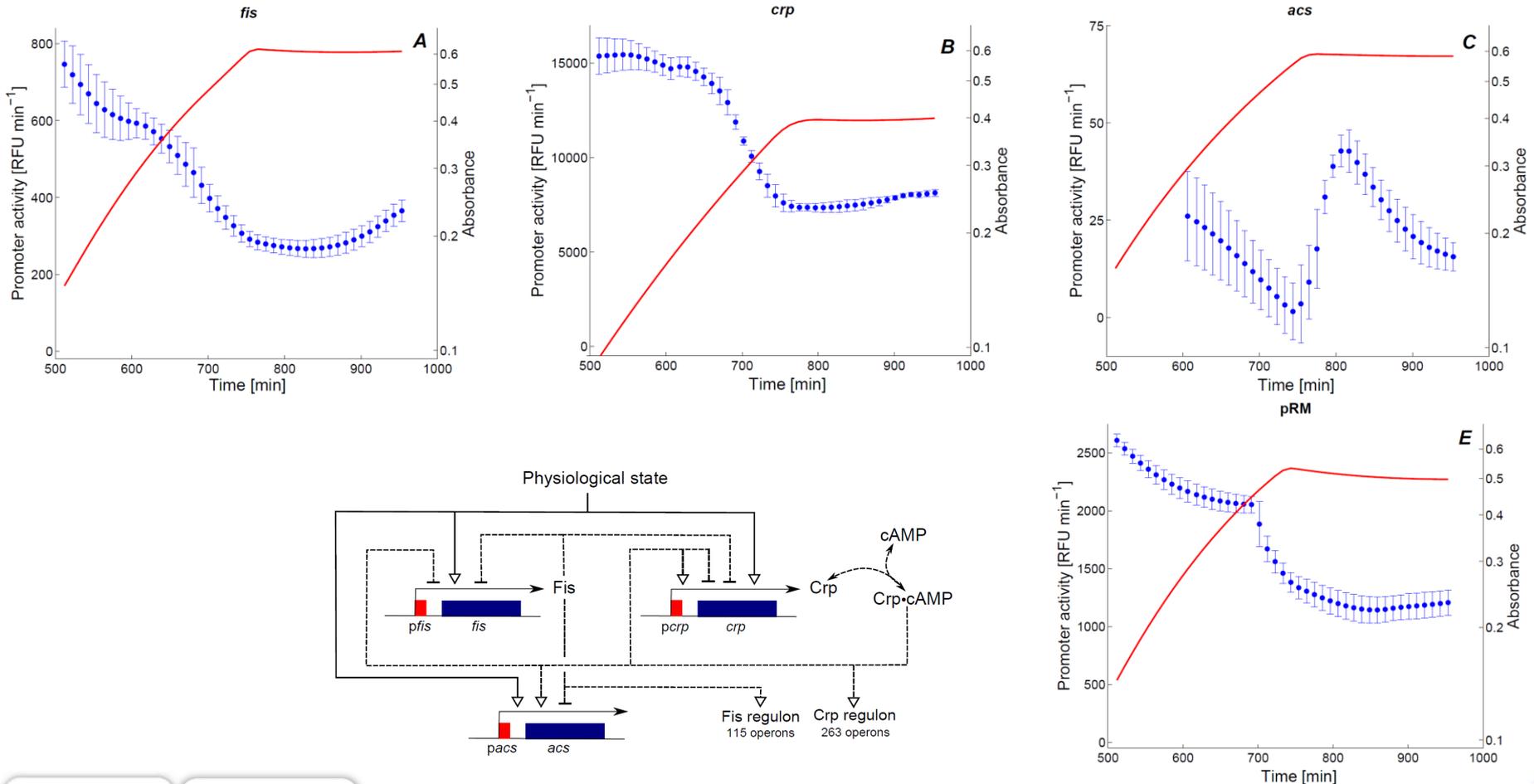
Growth rate



Promoter activity

# Real-time monitoring of gene expression

- Monitoring of activity of *crp*, *fis*, *acs* and constitutive phage promoters during growth transition



# Bias introduced by plasmid copy number

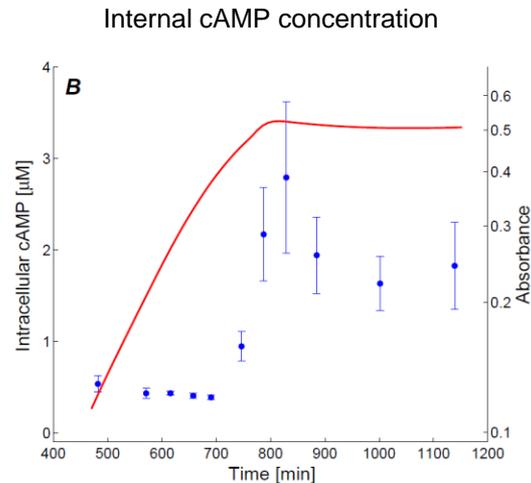
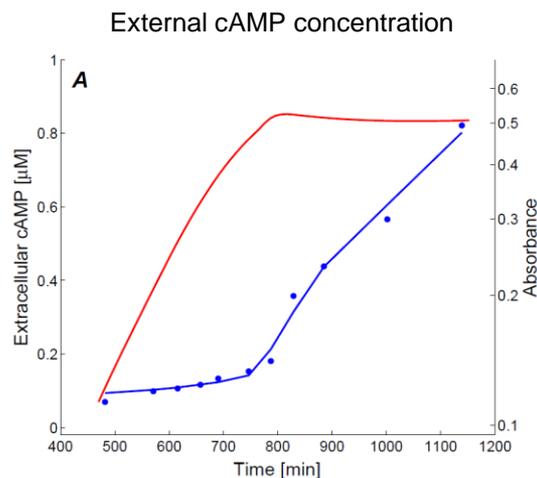
- Plasmids are relatively easy to construct and have strong signal, but ... **plasmid copy number** varies with growth rate

Lin-Chao and Bremer (1986), *Mol. Gen. Genet.*, 203(1):143-9

- Measurement of relative plasmid copy number using qPCR
- Variation in plasmid copy number preserves qualitative shape of profiles, but introduces quantitative bias
- **Conclusion:** need for analysis method that corrects for growth-phase dependent variations of plasmid copy number

# Measurement of cAMP

- Measurement of cAMP concentration during growth transition:
  - Measurement of extracellular cAMP concentration
  - Development of kinetic model accounting for cAMP import/export
  - Determination of intracellular cAMP concentration from measurements and model



- Good correspondence with intracellular cAMP profiles published in literature

Kao et al. (2004), *Proc. Natl. Acad. Sci. USA*, 101(2):641-6

# Approach

- **Real-time monitoring of dynamic response** of network to depletion of carbon source (glucose):
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# Model of promoter activities

- Simple **model of promoter activity** separating specific effects of transcription factors from global effect of physiological state

$$p(t) = k p_1(t) p_2(t)$$

$k$  : maximum promoter activity

$p_1(t)$  : regulation by global physiological state

$p_2(t)$  : regulation by specific transcription factors

$p_1(t)$  and  $p_2(t)$  vary between 0 and 1

# Model of promoter activities

- Simple **model of promoter activity** separating specific effects of transcription factors from global effect of physiological state

$$p(t) = k p_1(t) p_2(t)$$

- Normalization with respect to **reference state** at  $t^0$  to get rid of unknown constant  $k$  and logarithmic transformation:

$$\log \frac{p(t)}{p^0} = \log \frac{p_1(t)}{p_1^0} + \log \frac{p_2(t)}{p_2^0}$$

Convenient choice of reference state: growth arrest (expression peak of *acs*) or steady state after growth transition

# Model of promoter activities

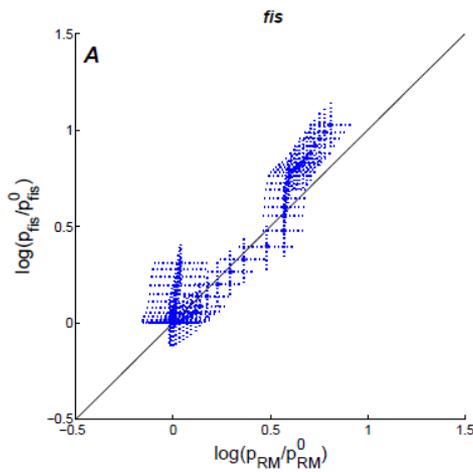
- **Hypothesis 1:** effect of global physiological state (measured by phage promoter) is dominant and effect of specific regulators is negligible ( $p_2(t) \approx p_2^0$ ):

$$\log \frac{p(t)}{p^0} = \log \frac{p_{RM}(t)}{p_{RM}^0}$$

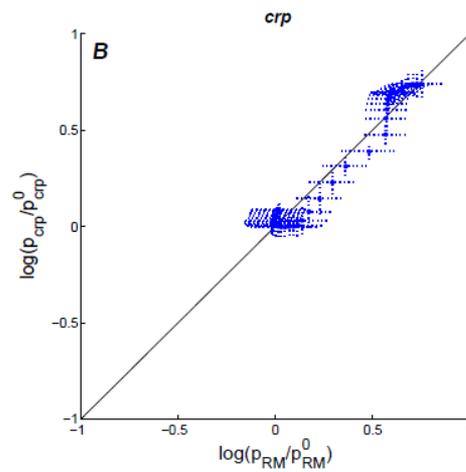
- Advantages of model:
  - Straightforward to test by means of experimental data
  - Non-parametric, does not require model calibration
  - No effect of plasmid copy number variation if promoter activity is measured in same plasmid vector

# Test of hypothesis 1

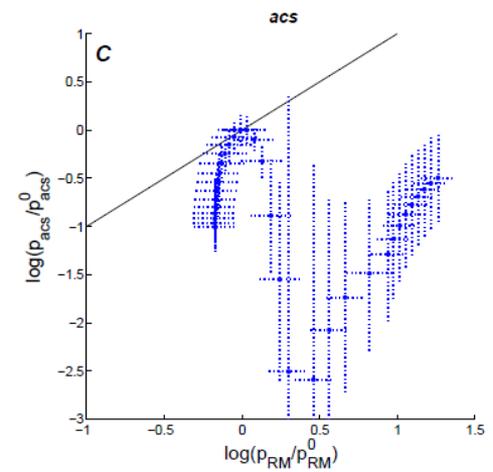
- Global effect is dominant for expression control of transcription factors (*crp* and *fis*), but not for metabolic gene (*acs*)



$$R^2 = 0.93$$



$$R^2 = 0.96$$



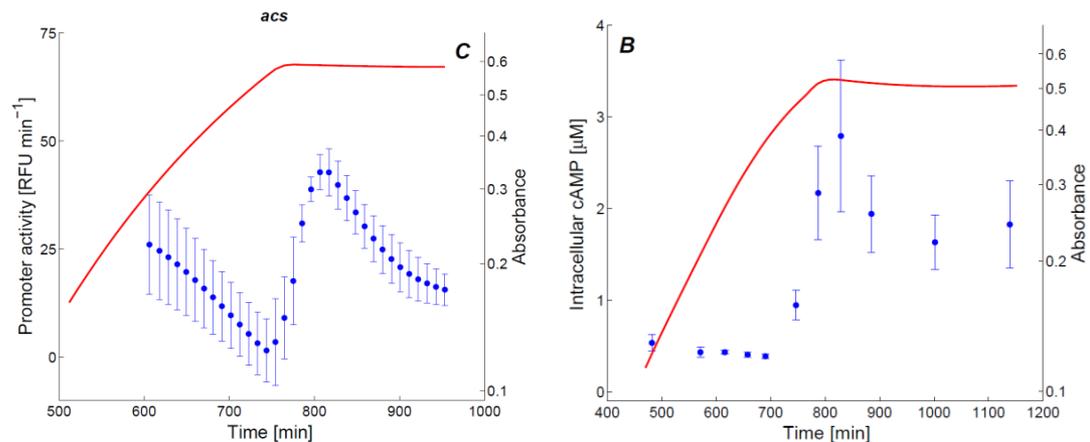
$$R^2 = 0.08$$

# Model of promoter activities

- **Hypothesis 2:** effect of specific regulators is not negligible and can be reduced to effect of change in cAMP concentration  $c(t)$ :

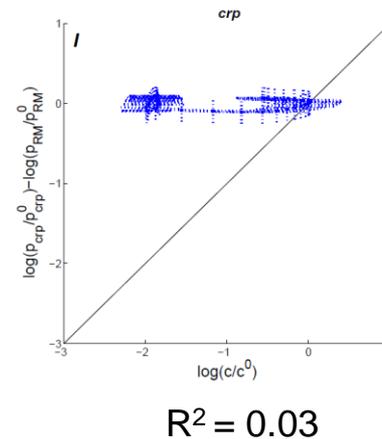
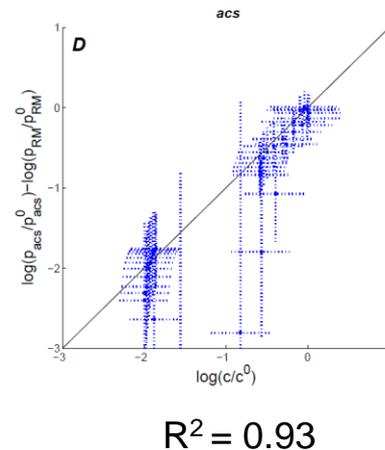
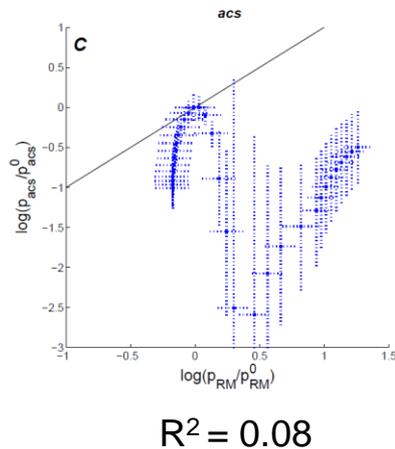
$$\log \frac{p(t)}{p^0} - \log \frac{p_{RM}(t)}{p_{RM}^0} = \log \frac{c(t)}{c^0}$$

- Hypothesis based on data, but biological assumptions underlying simplification can be explicitly formulated



# Test of hypothesis 2

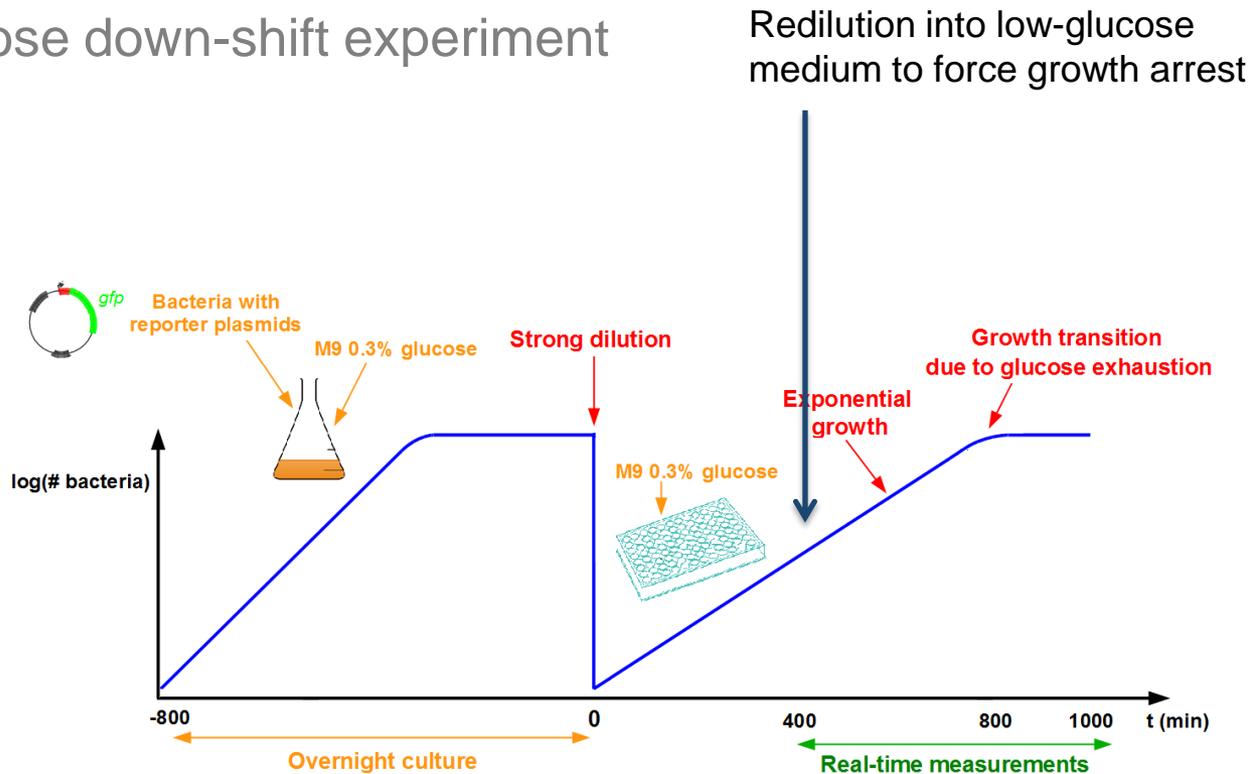
- Combination of global effect and specific effect of cAMP explains variation in *acs* promoter activity



- Addition of cAMP as regulator yields bad fit for *crp* and *fis*: no improvement upon simpler hypothesis 1

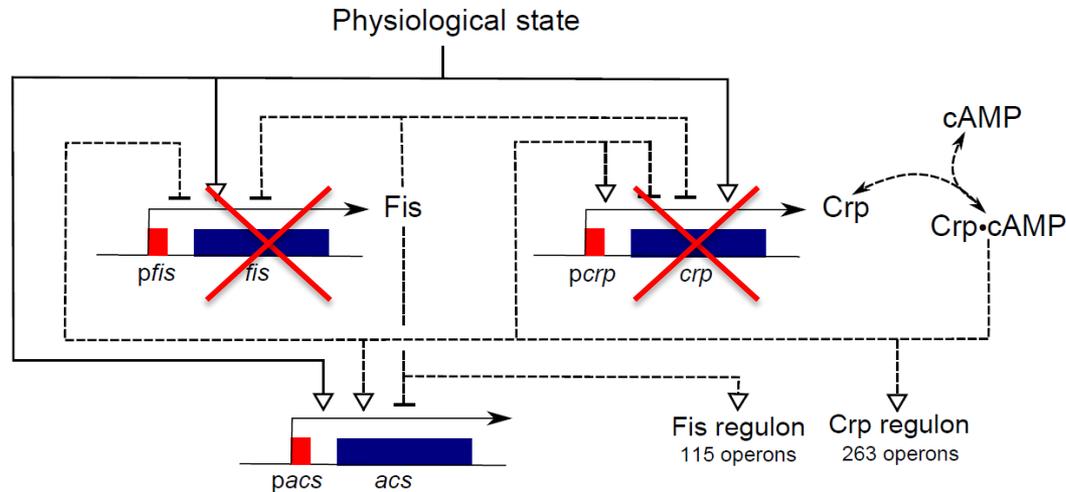
# Other experimental conditions

- Experiments and model tests were repeated in other conditions:
  - Glucose down-shift experiment



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  - Glucose down-shift experiment
  - Deletion mutant *crp*
  - Deletion mutant *fis*



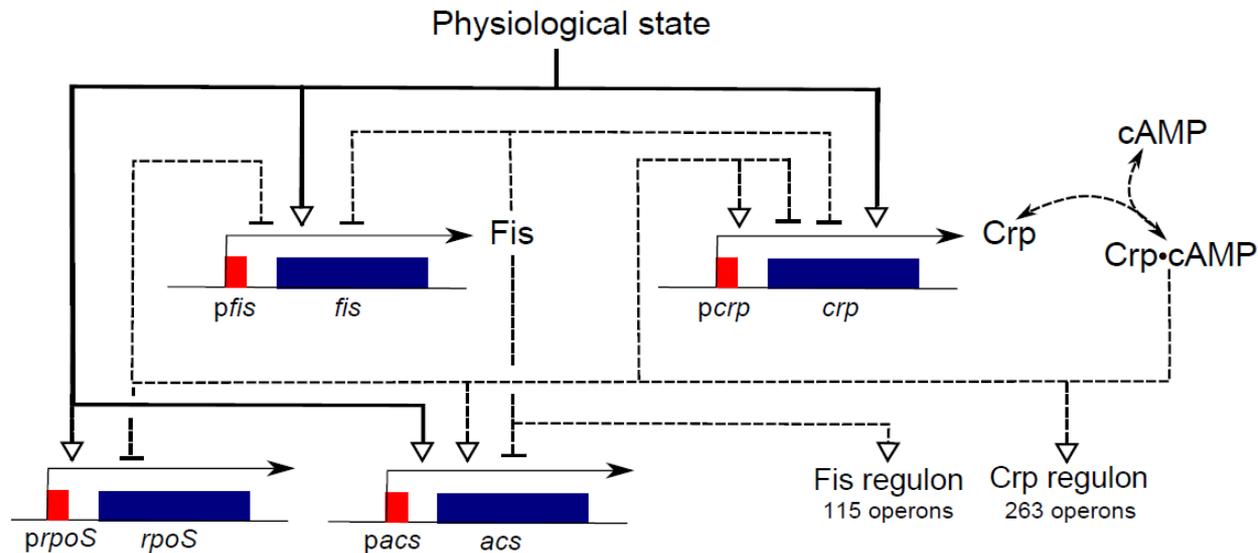
# Other experimental conditions

- Experiments and model tests were repeated in other conditions:
  - Glucose down-shift experiment
  - Deletion mutant *crp*
  - Deletion mutant *fis*
- Additional data confirm conclusions:
  - Effect of global physiological state dominant for transcriptional control of genes encoding transcription factors Fis and Crp
  - Combined effect of global physiological state and cAMP accounts for variation of promoter activity of *acs*

# Other regulators

- Is effect of global physiological state also dominant in transcriptional control of other regulators?

RpoS ( $\sigma^S$ ), master stress regulator in *E. coli*, inhibited by Crp-cAMP

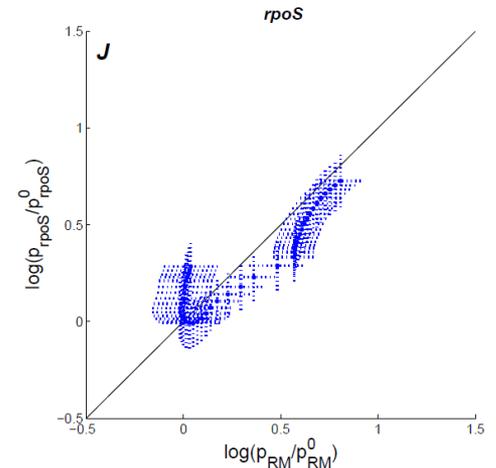
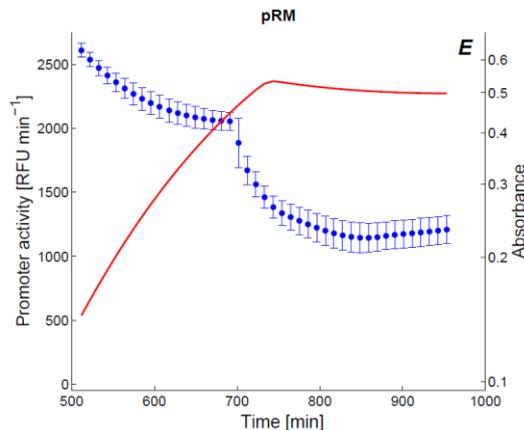
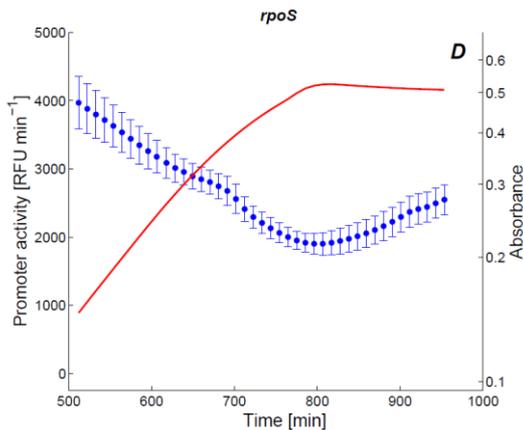


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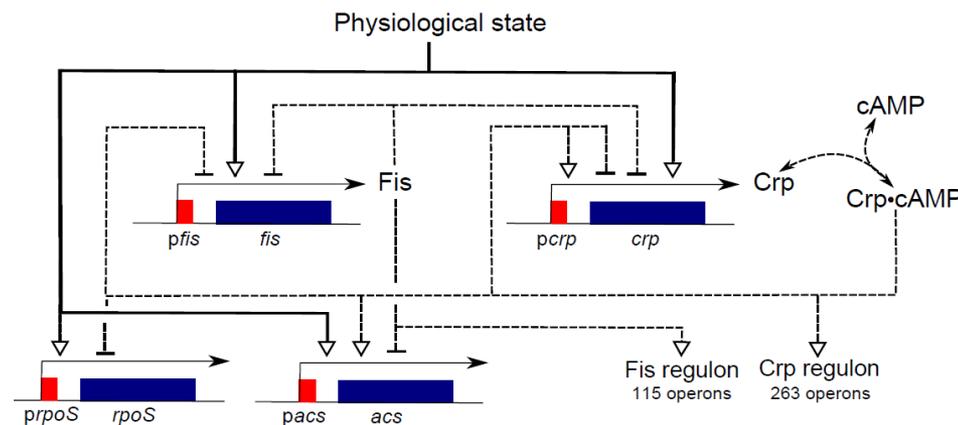
- Test of hypothesis 1 in different conditions confirms dominant role of global physiological state



$$R^2 = 0.84$$

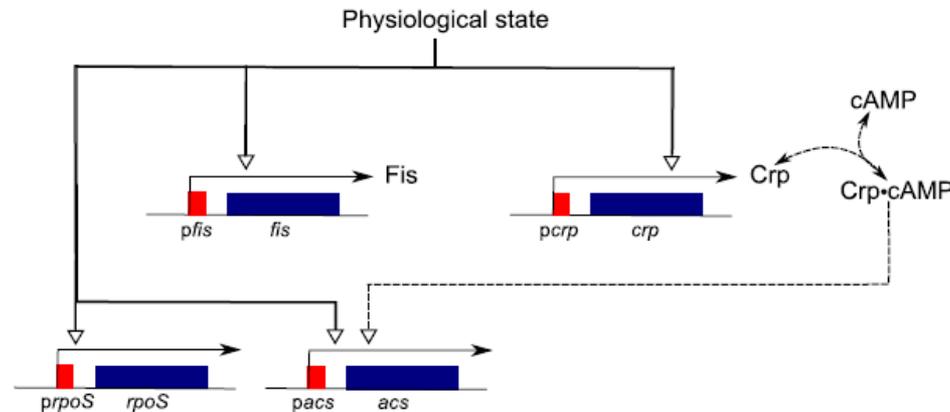
# Conclusions

- Control of gene expression across growth phases is shared between global physiological state and transcription factors
- Method to dissect shared control of promoter :
  - Simple mathematical model of promoter activity
  - Carefully designed data analysis procedures
- Application of method to analysis of regulatory circuit involving key regulators of carbon metabolism in *E. coli*



# Conclusions

- Transcriptional control of genes encoding transcription factors is dominated by growth-phase-dependent effect
  - Many regulatory interactions involving Crp·cAMP and Fis do not contribute to transcriptional control in our conditions
  - Choice of growth conditions? Weak effects?



Berthoumieux *et al.* (2013), *Mol. Syst. Biol.*, 9:634

# Conclusions

- Results question central role often attributed to transcriptional regulatory networks
- **Alternative view:** specific effects complement and finetune global control exerted by physiological state, notably gene expression machinery

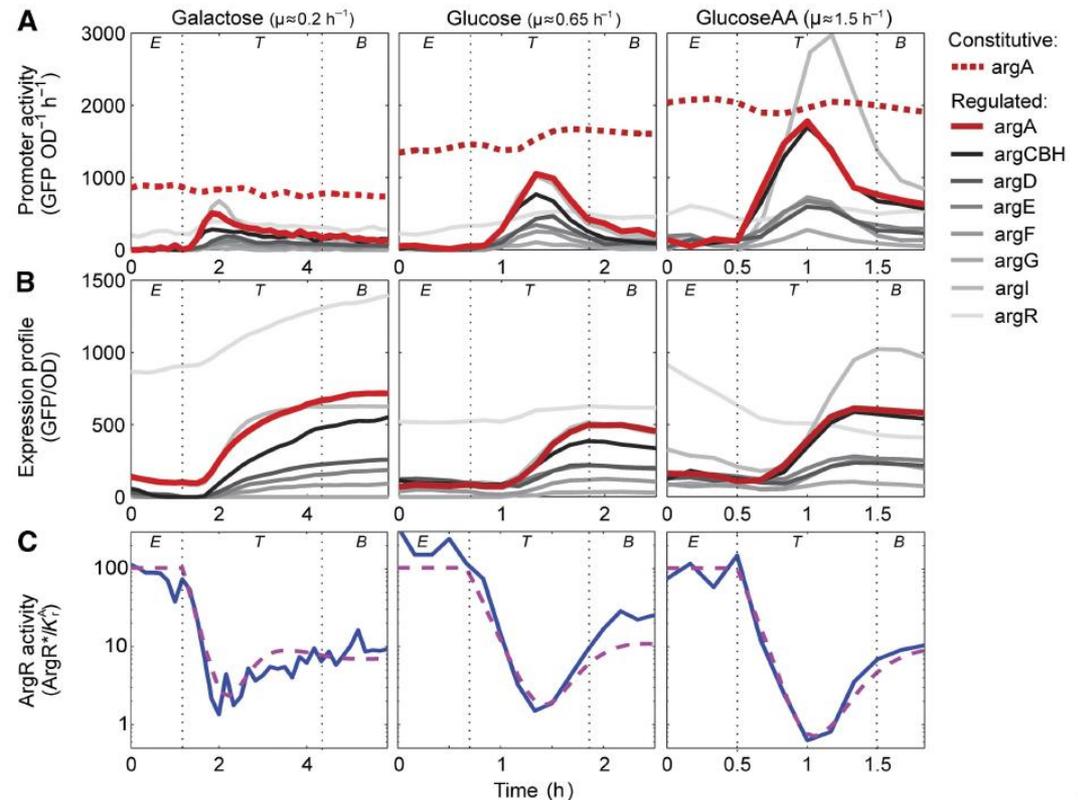
Gerosa *et al.* (2013), *Mol. Syst. Biol.*, 9:658

- Consequences for interpretation of transcriptome data and design of synthetic circuits



# Conclusions

- Analysis generalizable to other networks?
  - Dissection of control of arginine biosynthesis genes in *E. coli*
  - Control of gene expression is combination of global physiological state and specific effects (arginine concentration)

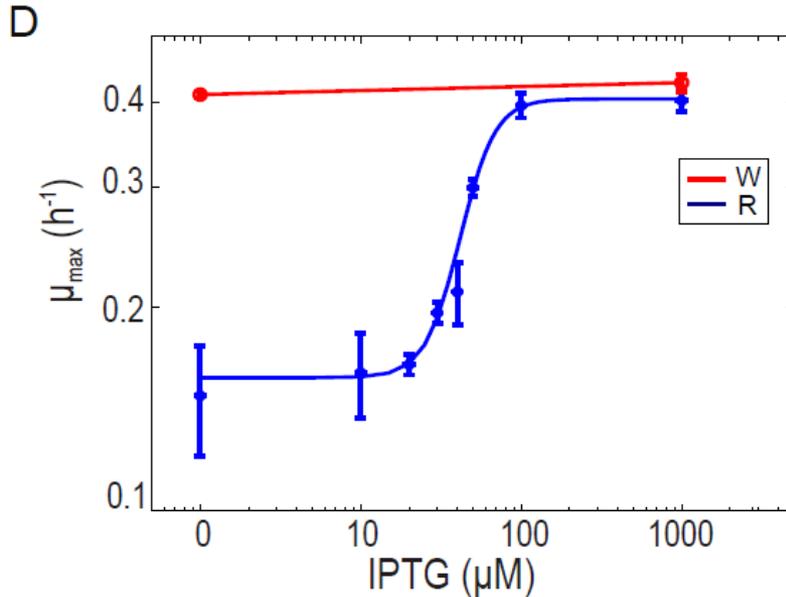


Gerosa *et al.* (2013), *Mol. Syst. Biol.*, 9:658

# Perspectives

- Can we control global physiological state, and thus gene expression program of the cell?

Engineering of *E. coli* genome to put transcriptional machinery under control of inducible promoter



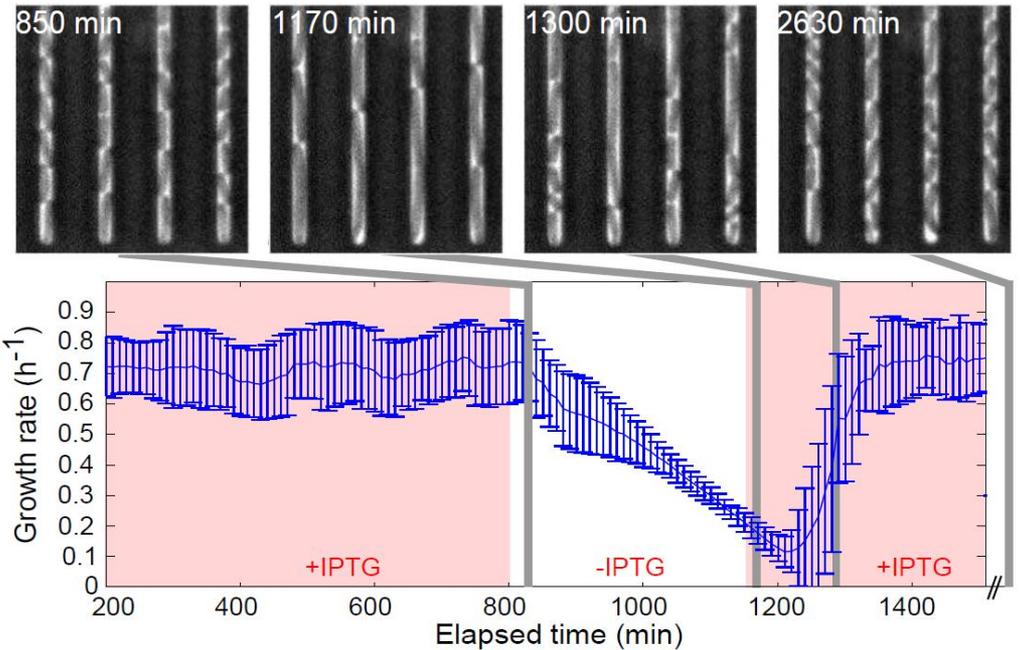
Izard *et al.*, submitted for publication

- Finetuning of growth rate...

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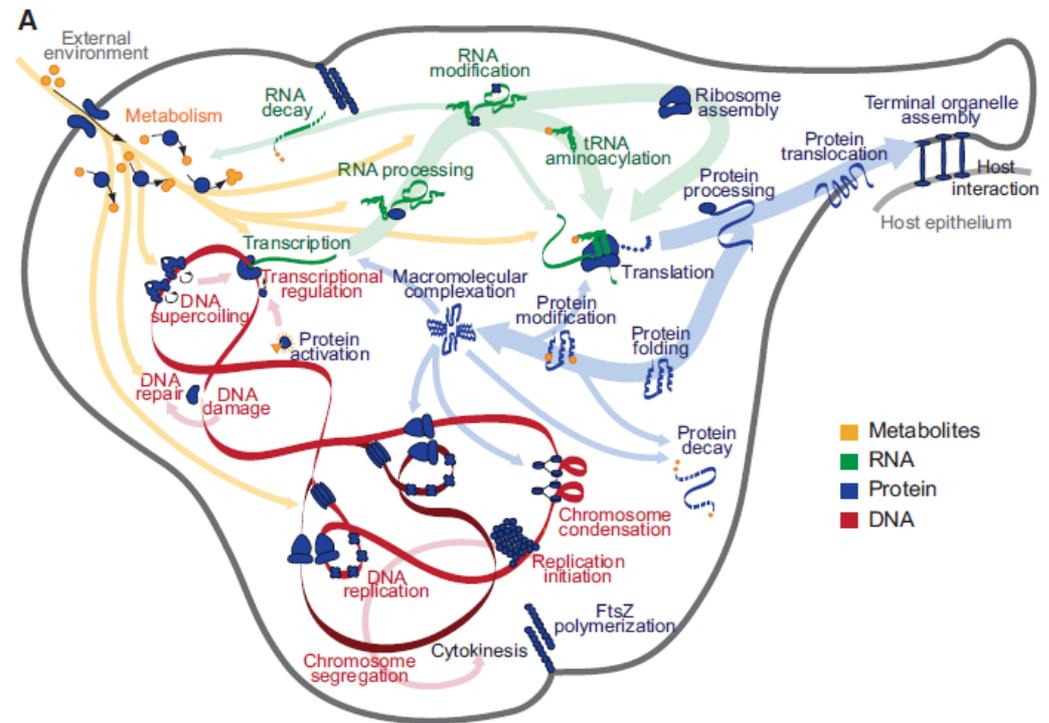
Izard *et al.*, submitted for publication  
Collaboration with A. Lindner

- Finetuning of growth rate ... in reversible way

# Whole-cell model *M. genitalium*

- Metabolic networks are integrated with gene networks and signalling networks

Complex multi-level system with feedback across different time-scales

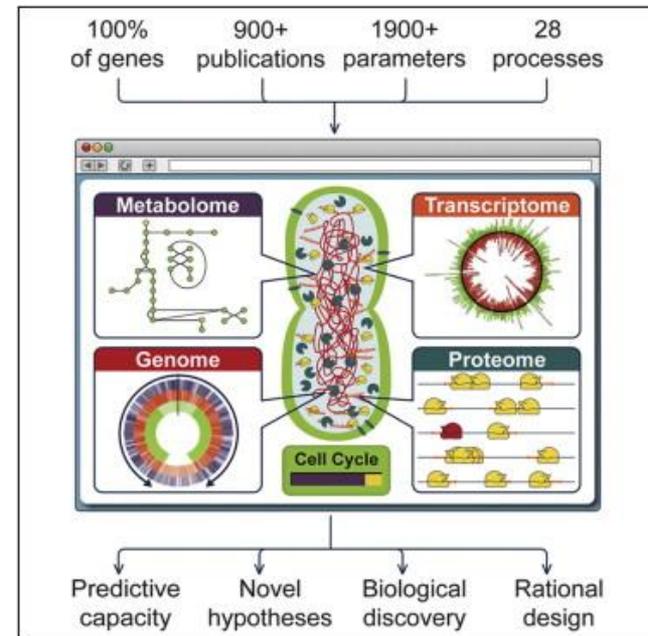


Whole-cell model of *Mycoplasma genitalium*

Karr *et al.* (2012), *Cell*, 150(2): 389-401

# Whole-cell model *M. genitalium*

- Whole-cell model represents huge modelling effort:
  - Whole-genome model including **complete** known metabolic, gene, and signalling networks

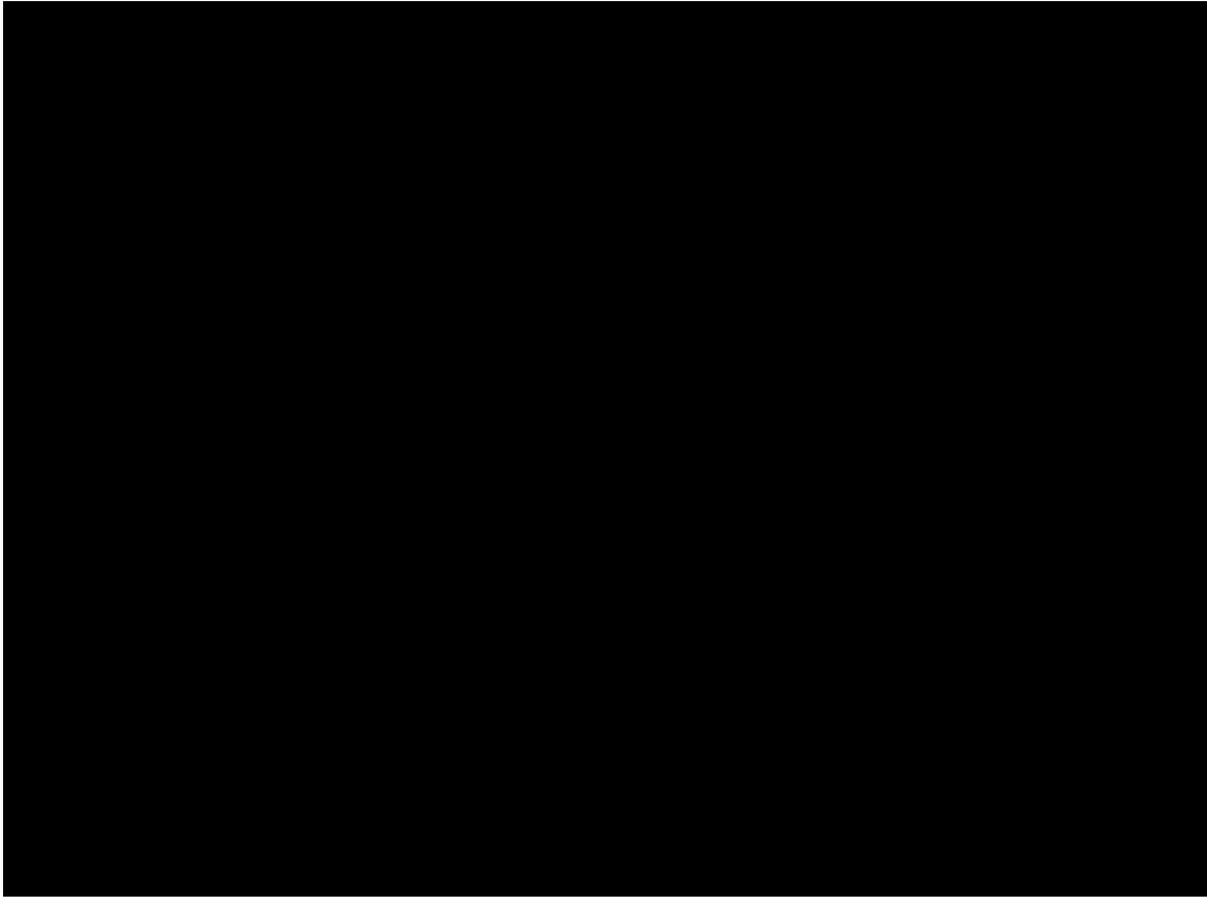


Karr *et al.* (2012), *Cell*, 150(2): 389-401

- Variety of **formalisms** to model the 28 modules: FBA, kinetic ODE models, Boolean models, Markov chains, ...
- Cell cycle simulated for >100 cells, >30 mutants on 128-core machine

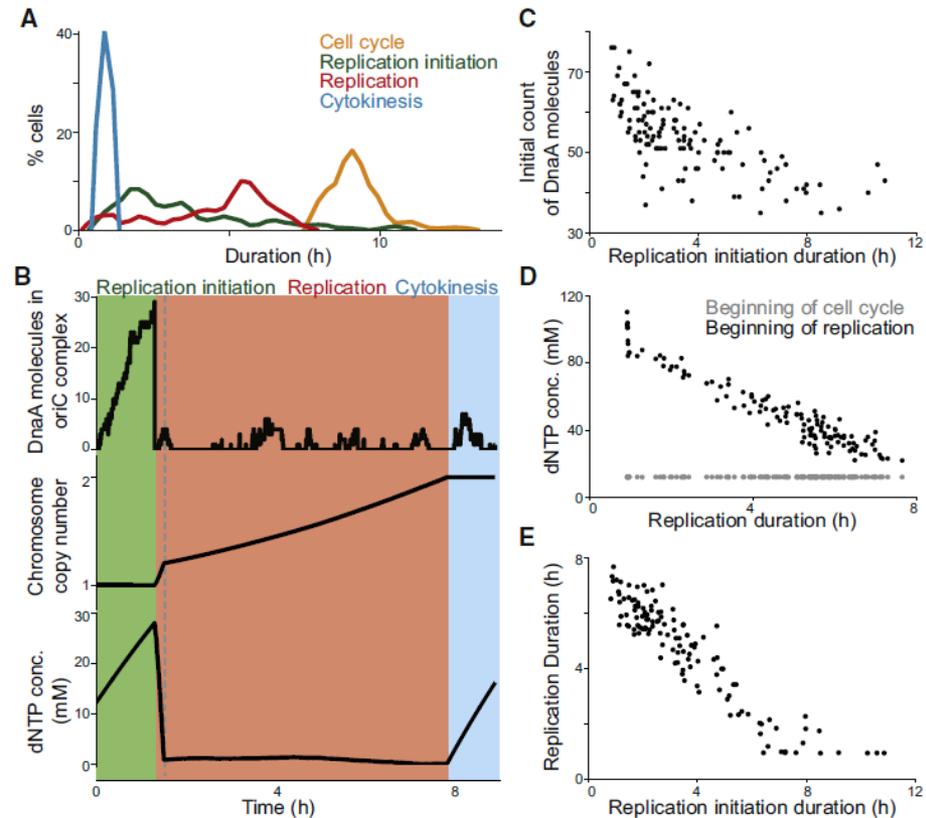
# Whole-cell model *M. genitalium*

- Whole-cell simulation of *M. genitalium* cell cycle



# Whole-cell model *M. genitalium*

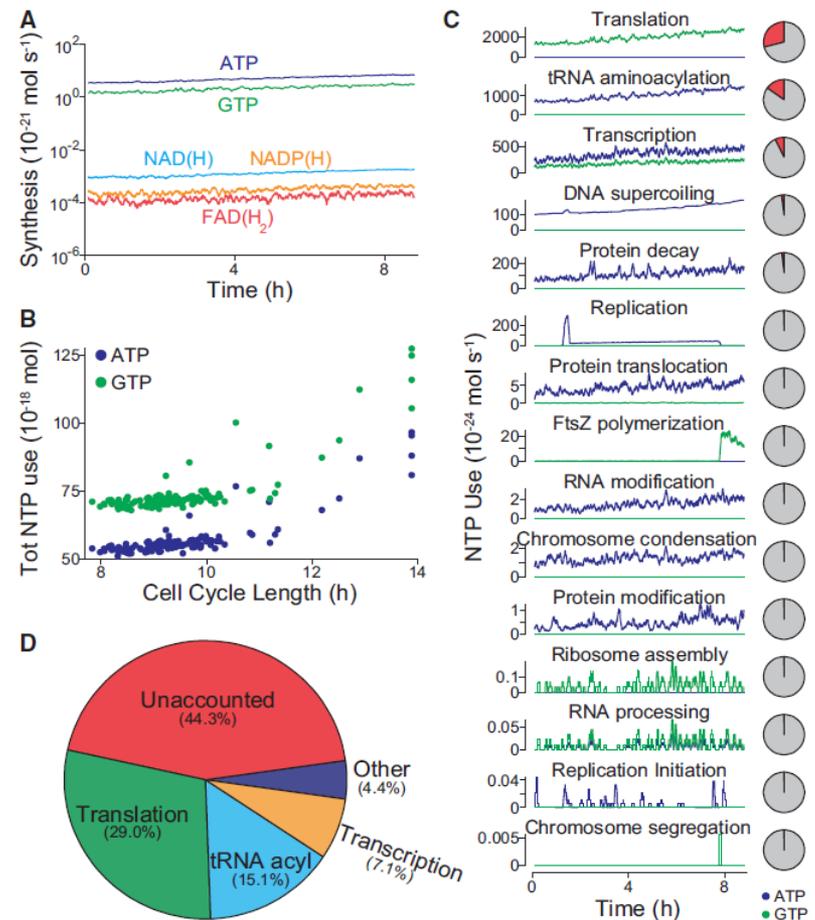
- Whole-cell simulations have provided new insights into **robustness of cell-cycle duration**
  - High variability of replication initiation buffered by dNTP-dependent duration of replication
  - This metabolic control of replication leads to decreased variability of cell-cycle length



Karr *et al.* (2012), *Cell*, 150(2): 389-401

# Whole-cell model *M. genitalium*

- Whole-cell simulations have provided new insights into **global use and allocation of energy**
  - Transcription and translation most costly processes
  - Energy use largely independent of cell-cycle length
  - Usage of almost half of produced energy not accounted for!



Karr *et al.* (2012), *Cell*, 150(2): 389-401

# Lack of quantitative information: strategies

- Three main strategies to deal with lack of quantitative data:
  - **Test of parameter sensitivity**
  - Model reduction and simplification
  - Parameter estimation from time-series data

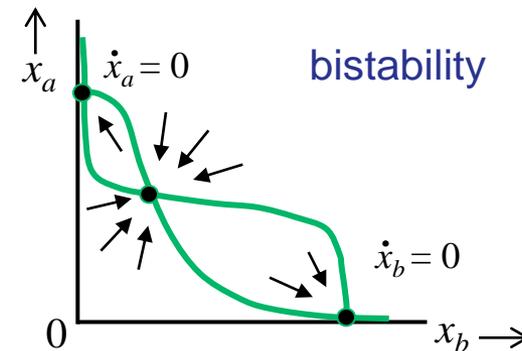
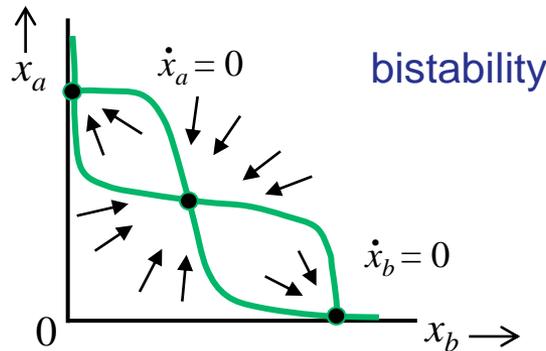
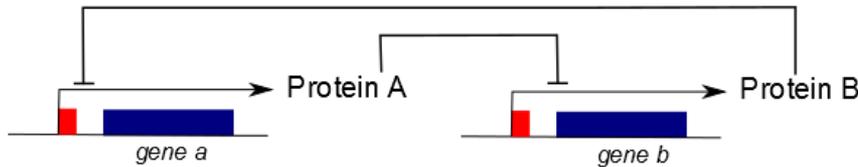
De Jong and Ropers (2006), *Brief. Bioinform.*, 7(4):354-363

# Test of parameter sensitivity

- Important dynamic properties are expected to be **robust** over large ranges of parameter values

Important dynamic properties should be insensitive to moderate variations in parameter values

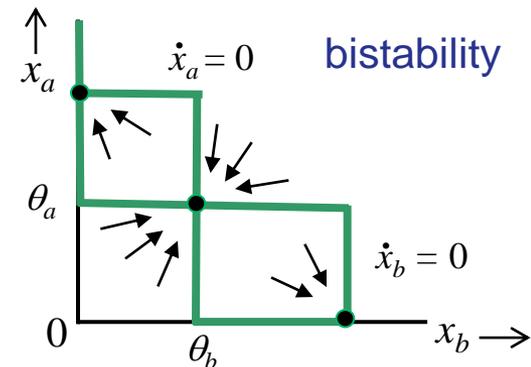
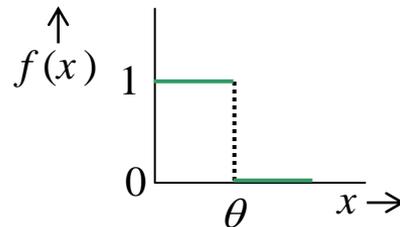
Stelling *et al.* (2004), *Cell*, 118(6):675-685



# Model reduction and simplification

- Use model reduction and simplification to obtain models that can be analyzed with less information on parameter values
  - Piecewise-linear instead of nonlinear models
  - Also: Boolean models

$$\begin{aligned}\dot{x}_a &= \kappa_a f(x_b) - \gamma_a x_a \\ \dot{x}_b &= \kappa_b f(x_a) - \gamma_b x_b\end{aligned}$$



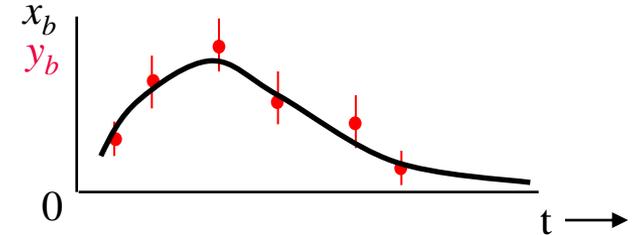
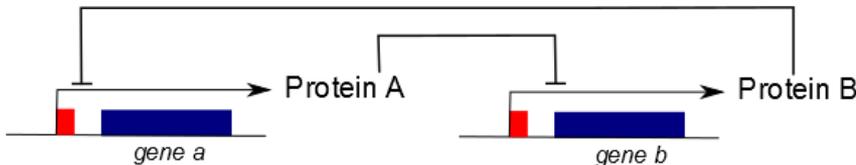
Glass and Kauffman (1973), *J. Theor. Biol.*, 39(1):103-29  
de Jong et al. (2004), *Bull. Math. Biol.*, 66(2):301-40

# Parameter estimation

- **Estimate** parameter values from experimental time-series data  
**Systems identification** in control and engineering

Ljung (1999), *System Identification: Theory for the User*, Prentice Hall

- Given model structure, search parameter values for which model predictions best fit experimental data



- Minimization of objective function, for instance sum of squared errors:  
$$\sum_t (x(t, \theta) - y(t))^2$$

Possibility to add constraint or penalty terms to restrict parameter space

# Lack of quantitative information: strategies

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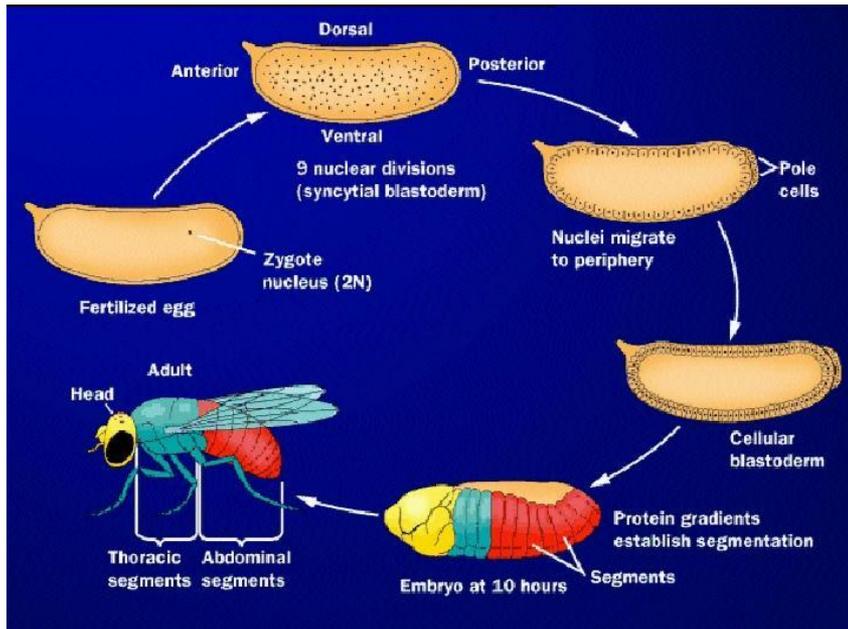
- Illustration: models of developmental processes in multicellular organisms

Development of *Drosophila* embryo



# Development of *Drosophila* embryo

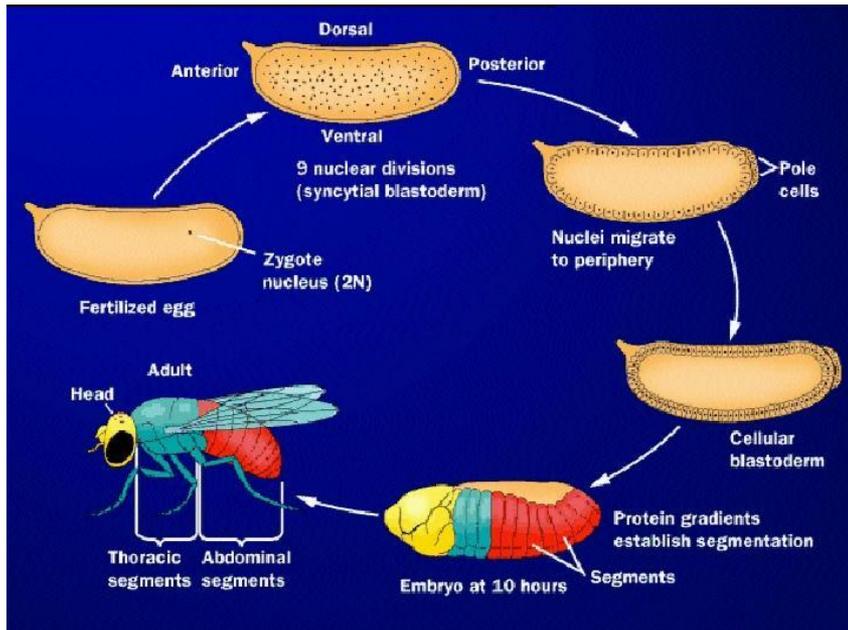
- Development of *Drosophila melanogaster* (fruit fly)



Purves *et al.* (1998), *Life: The Science of Biology*, Sinauer

# Development of *Drosophila* embryo

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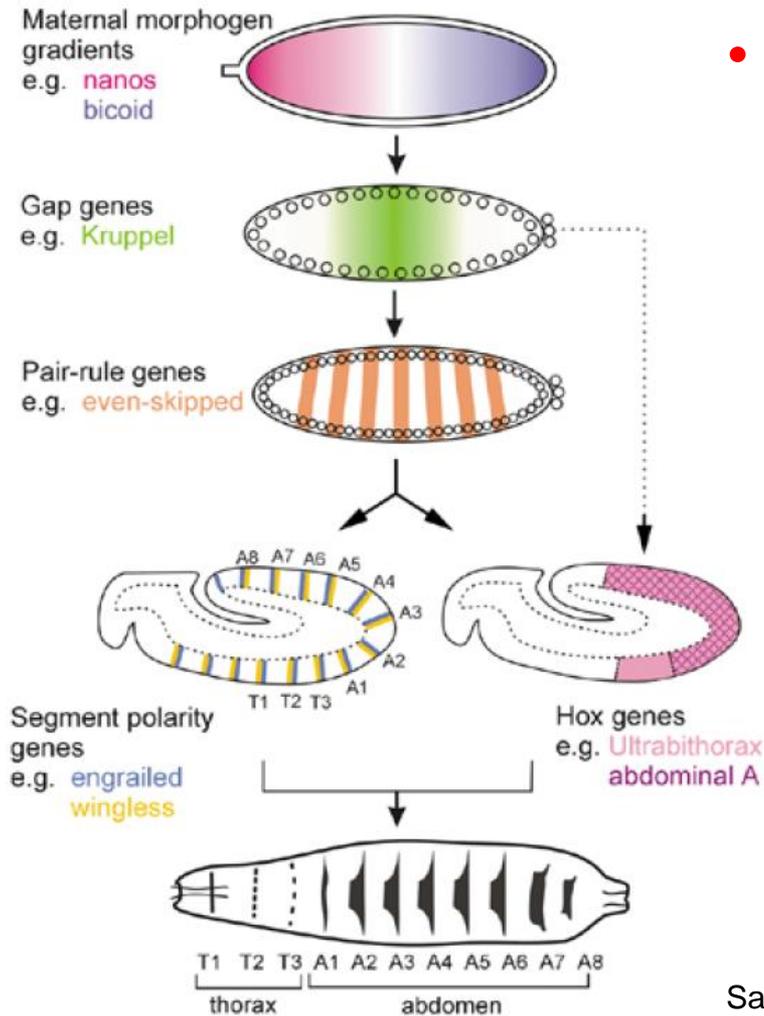


Purves *et al.* (1998), *Life: The Science of Biology*, Sinauer

Tomer *et al.* (2012), *Nat. Methods*, 9(7):755–63

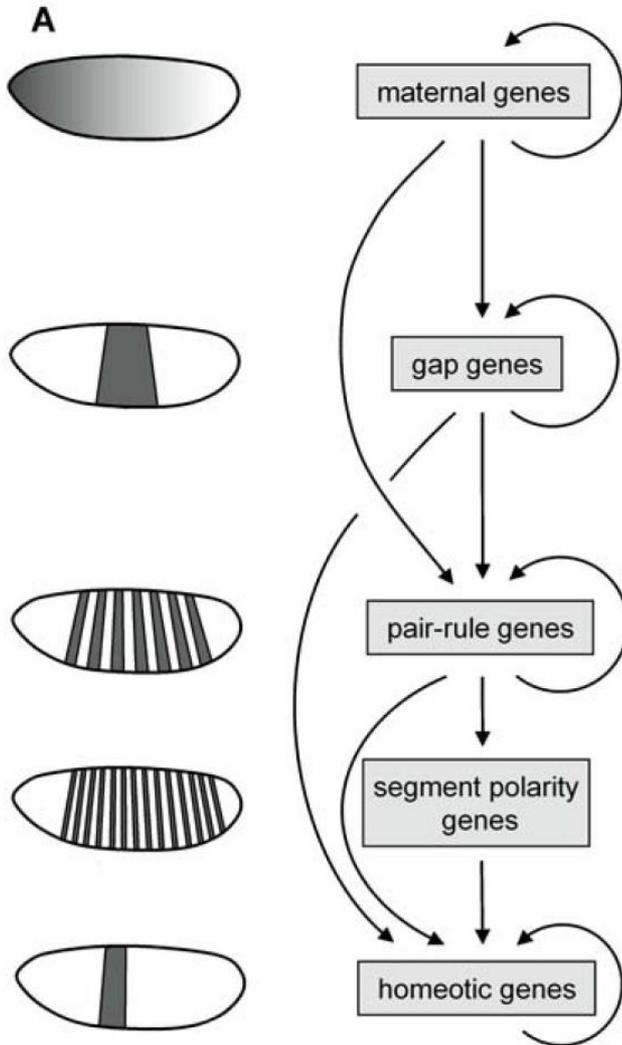
# Development of *Drosophila* embryo

- Spatiotemporal gene expression patterns during early development of *Drosophila* (fruit fly)



Sanson (2001), *EMBO Rep.*, 2(12):1083–8

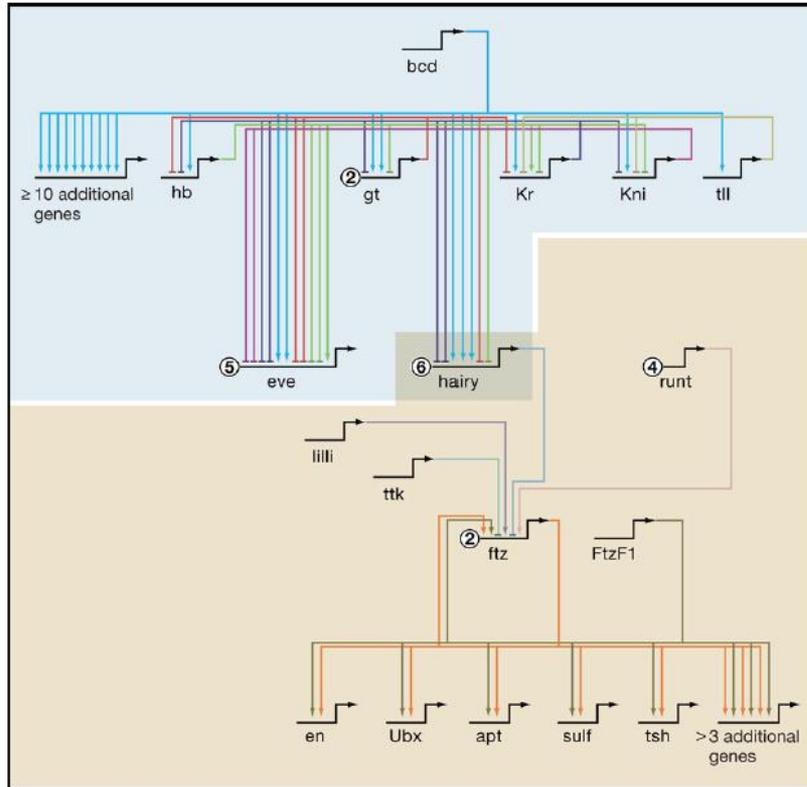
# Development of *Drosophila* embryo



- Spatiotemporal gene expression patterns during early development of *Drosophila* (fruit fly)
- Gene classes and their interactions responsible for establishment of gene expression patterns

Schroeder *et al.* (2004), *PLOS Biol.*, 4(2):e271

# Development of *Drosophila* embryo

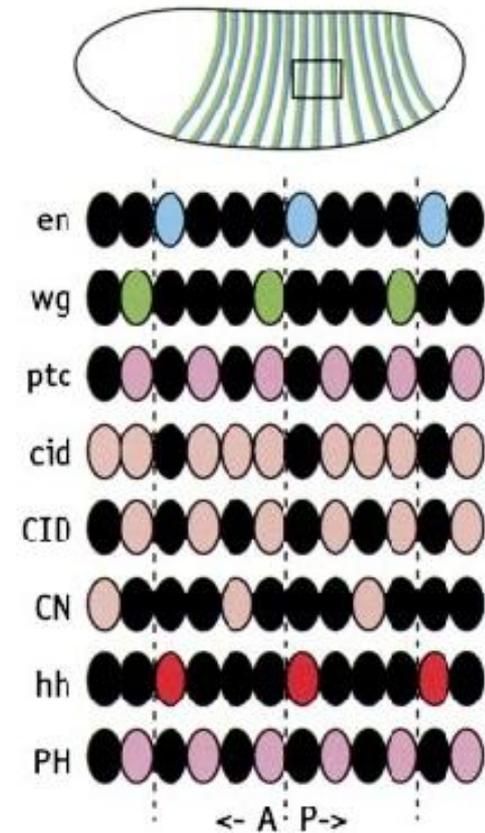
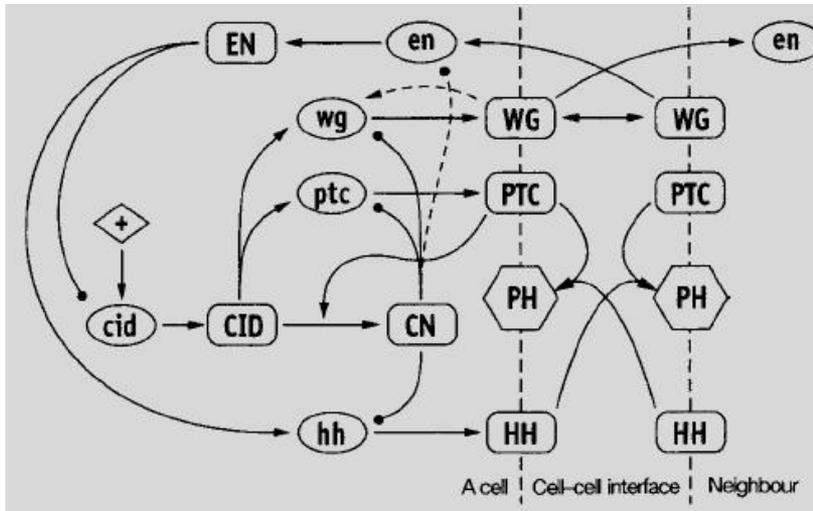


- Spatiotemporal gene expression patterns during early development of *Drosophila* (fruit fly)
- Gene classes and their interactions responsible for establishment of gene expression patterns
- Complex gene regulatory networks

Carroll (2008), *Cell*, 134(1):25-36

# Model of *Drosophila* segmentation

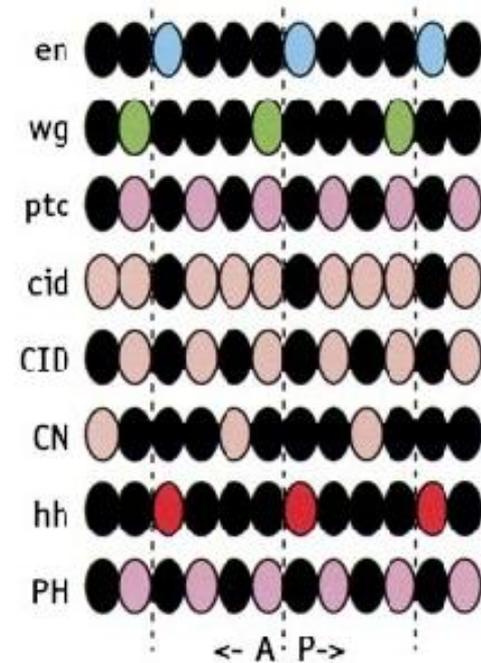
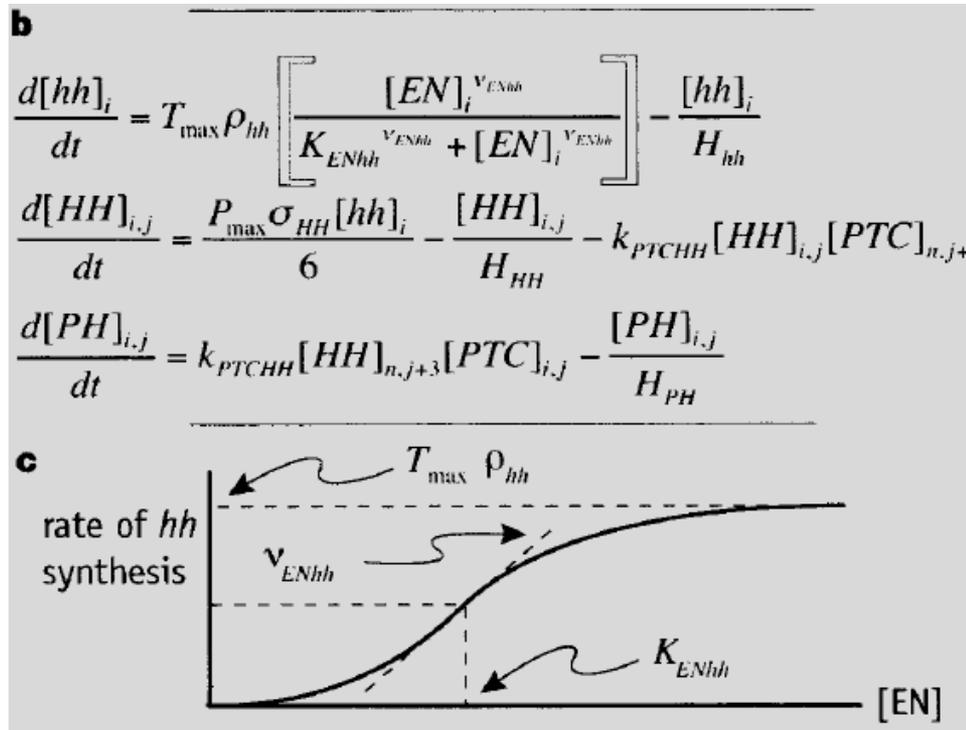
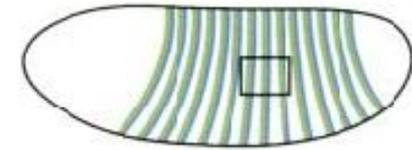
- Model of network of **segment polarity** genes in early development of *Drosophila*



von Dassow *et al.* (2000), *Nature*, 406(6792): 188-92

# Model of *Drosophila* segmentation

- Model of network of **segment polarity** genes in early development of *Drosophila*
  - 13 ODEs per cell and 48 parameters



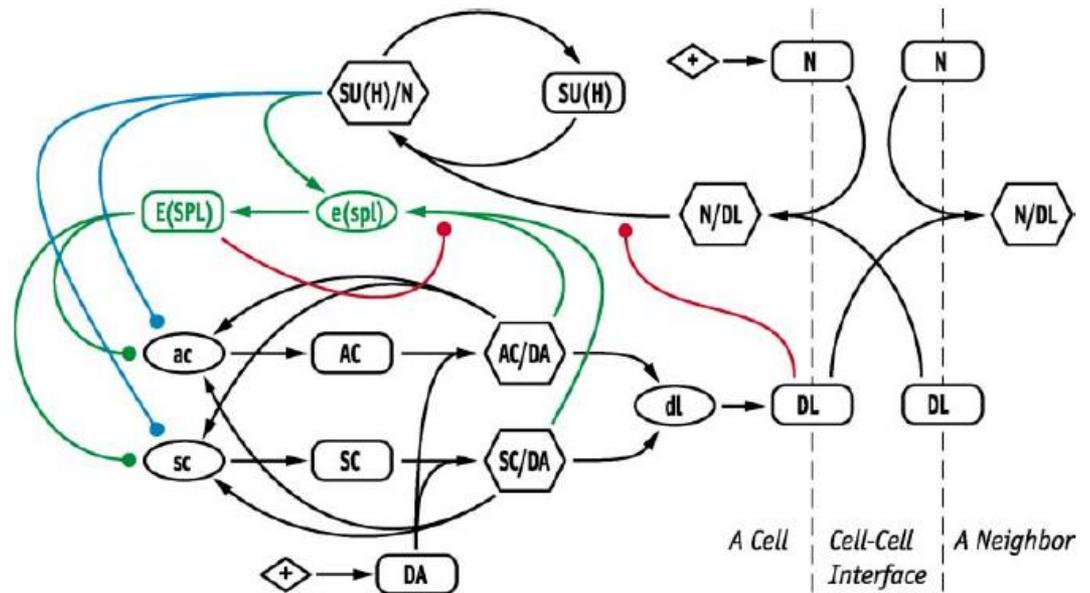
von Dassow *et al.* (2000), *Nature*, 406(6792): 188-92



# Robustness of gene expression patterns

- Robustness of model predictions to variations in parameter values confirmed for other developmental networks

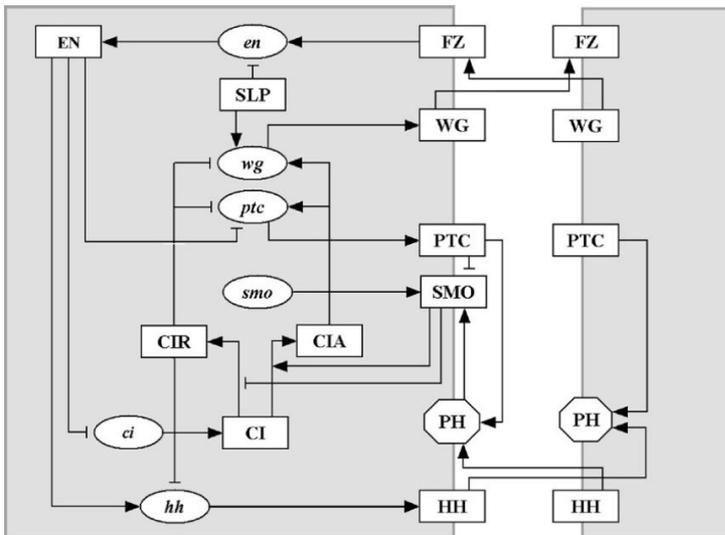
**Neurogenic network**, determining neuroblasts in embryos and sensory organ precursor cells in imaginary disks



Meir *et al.* (2002), *Curr. Biol.*, 12(10): 778-86

# Logical model of *Drosophila* segmentation

- **Logical model** of segment polarity network: variables take values 0/1 and Boolean functions to update variables



Albert and Othmer (2003), *J. Theor. Biol.*, 223(1):1-18

$hh_i$

$HH_i$

$ptc_i$

$PTC_i$

$PH_i$

$hh_i^{t+1} = EN_i^t$  and not  $CIR_i^t$

$HH_i^{t+1} = hh_i^t$

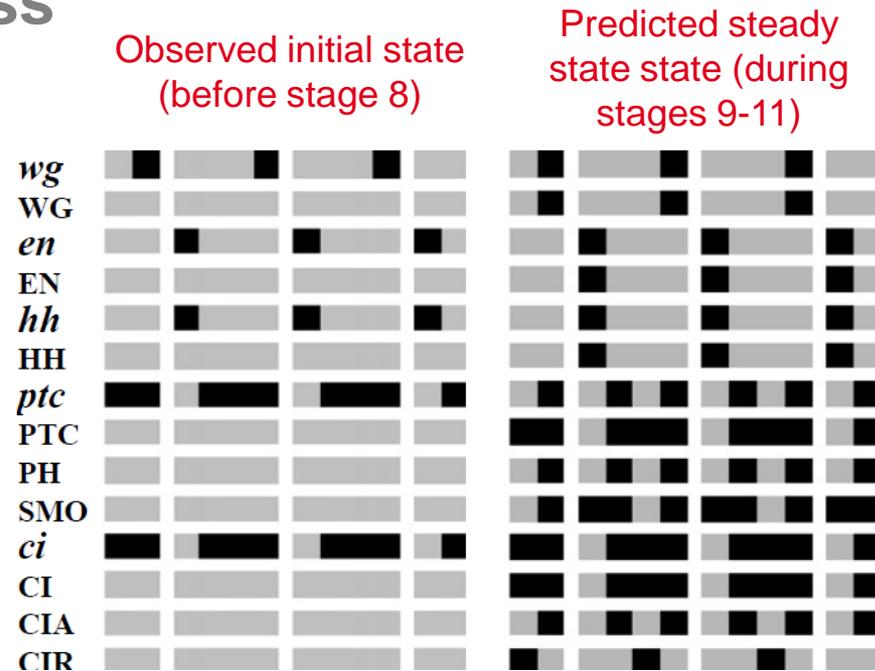
$ptc_i^{t+1} = CIA_i^{t+1}$  and not  $EN_i^t$  and not  $CIR_i^t$

$PTC_i^{t+1} = ptc_i^t$  or ( $PTC_i^t$  and not  $HH_{i-1}^t$  and not  $HH_{i+1}^t$ )

$PH_i^t = PTC_i^t$  and ( $HH_{i-1}^t$  or  $HH_{i+1}^t$ )

# Logical model of *Drosophila* segmentation

- **Logical model** of segment polarity network: variables take values 0/1 and Boolean functions to update variables
- Logical models are based on topology of network only (no parametrization), but are capable of reproducing experimental data: **robustness**



Albert and Othmer (2003), *J. Theor. Biol.*, 223(1):1-18

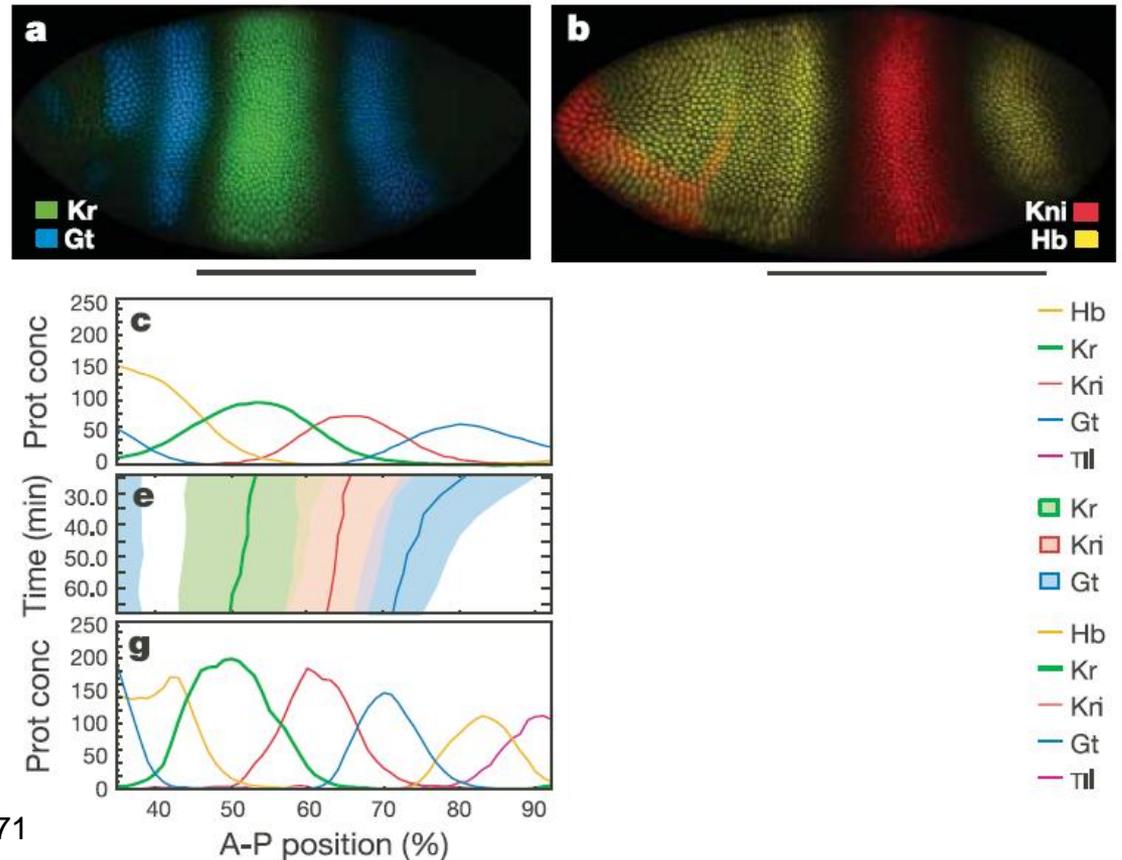
# Logical model of *Drosophila* segmentation

- **Logical model** of segment polarity network: variables take values 0/1 and Boolean functions to update variables
- Logical models are based on topology of network only (no parametrization), but are capable of reproducing experimental data: **robustness**
- **Generalized logical models** allow variables with several discrete values (more complicated update rules)

Sánchez *et al.* (2008), *Int. J. Dev. Biol.*, 52(1):1059-75

# Parameter estimation from *Drosophila* data

- Measurement of protein concentrations of gap genes during development of *Drosophila* embryo



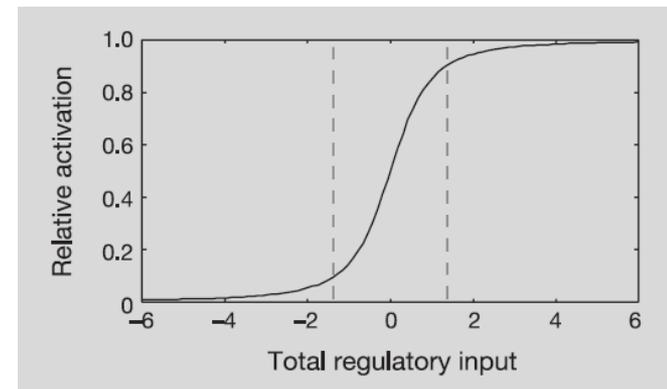
Jaeger et al. (2004), *Nature*, 430(6997):368-71

# Parameter estimation from *Drosophila* data

- Neural-network-like model of connections between gap genes
  - Model with 58 nuclei and 7 variables (proteins) per nucleus
  - Free diffusion of proteins because at early stages of development embryo is syncytium (multinucleate cell)
  - Sigmoidal response functions
  - Connectivity pattern encoded in parameter matrix  $T$ , so parametric **and** structural identification

$$\frac{dv_i^a}{dt} = R_a g(u^a) + D^a [(v_{i-1}^a - v_i^a) + (v_{i+1}^a - v_i^a)] - \lambda_a v_i^a$$

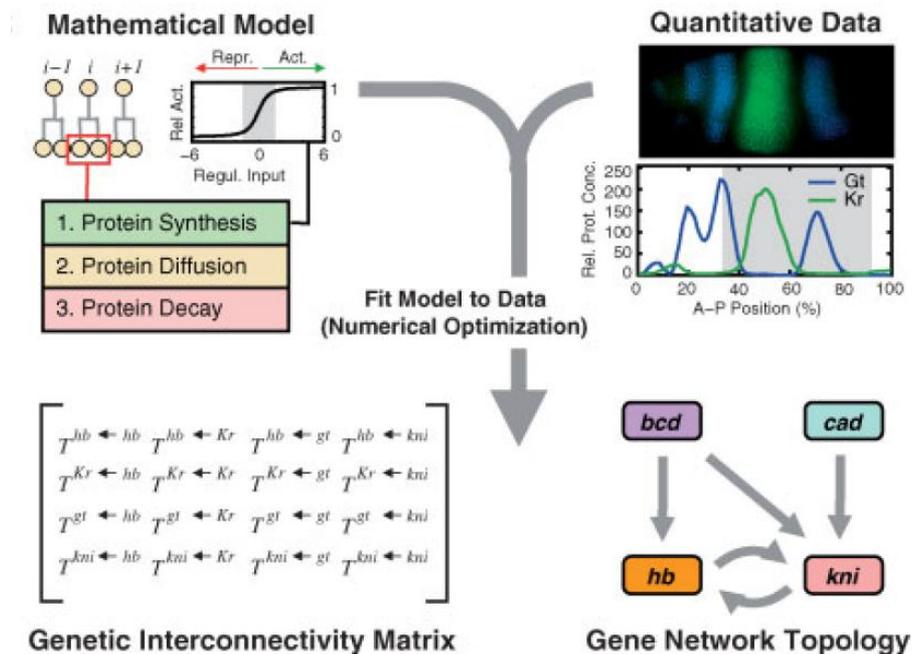
$$u^a = \sum_b T^{ab} v_i^b + m^a v_i^{Bcd} + h^a$$



Jaeger et al. (2004), *Nature*, 430(6997):368-71

# Parameter estimation from *Drosophila* data

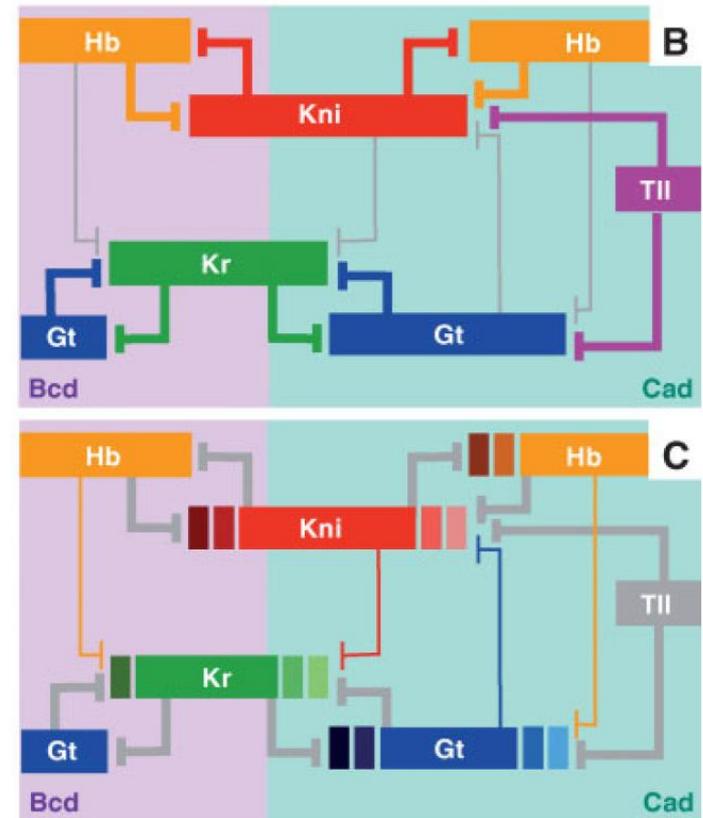
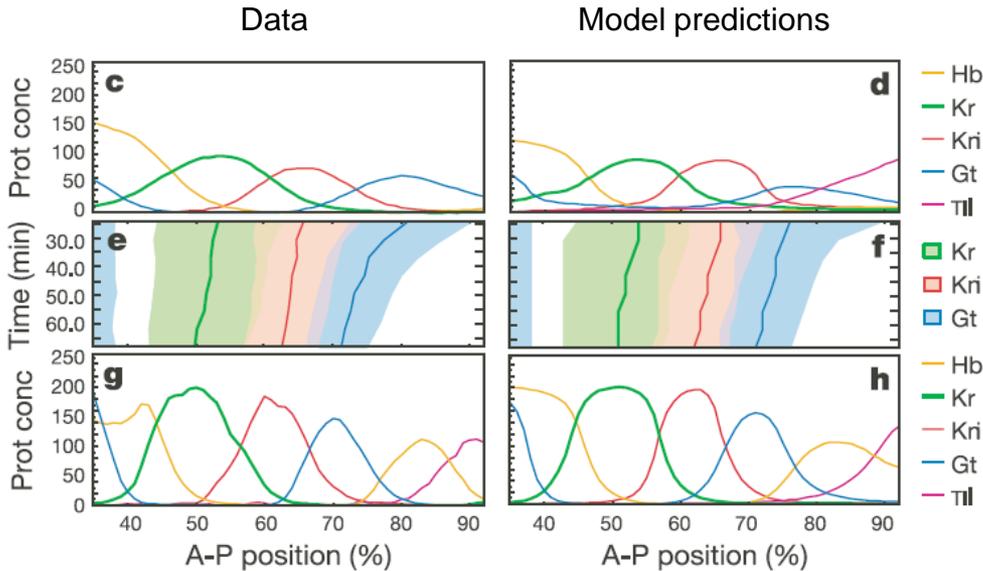
- Neural-network-like model of connections between gap genes
- Brute-force parameter estimation by fitting model to data  
Parallelized simulated annealing



Jaeger and Reinitz (2006), *BioEssays*,  
28(11):1102-11

# Shifts in gap gene domains

- What is function of **cross-inhibition** between gap genes?  
 Model predicts that they are important for shift in gap gene domains after their initial establishment



# Conclusions

- Several strategies to deal with lack of quantitative information
- Model predictions often robust to changes in parameter values and to simplification/reduction of equations
  - Model robustness reflects robustness of biological system?
- High-quality experimental data is becoming increasingly available, favoring estimation of parameter values from expression data
  - Quantitative models can make precise predictions of subtle dynamic phenomena

**Merci !**

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