

# Kinetic modeling of biochemical reaction networks

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September 25, 2023

# **MICROCOSME:** bacterial systems biology

 MICROCOSME: systems biology group at INRIA/Université Grenoble Alpes in Grenoble

Microbiologists, computer scientists, mathematicians, physicists, ...



https://team.inria.fr/microcosme



- **Objective:** analysis, engineering, and control of the growth of bacteria
  - Specific research problems shaped by **biological questions**
  - Problems often addressed by combination of models and experiments



#### **Overview**

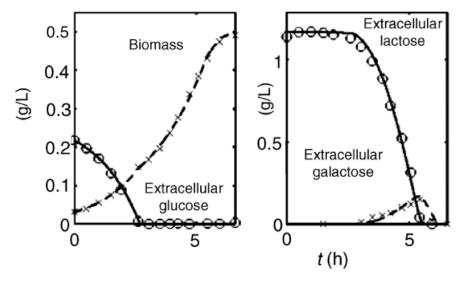
- Part 1. Systems biology and kinetic modeling
  - Introduction
  - Kinetic modeling of biochemical reaction networks
- Part 2. Metabolic network modeling
  - Kinetic modeling of metabolism
  - Metabolic control analysis (MCA)
  - Flux balance analysis (FBA)
  - Practical on flux balance analysis (COBRA)
- Part 3. Gene regulatory network modeling



## **Bacterial growth and metabolism**

 Bacterial metabolism is flexible, allowing cells to grow on different carbon sources

Preferential utilisation: diauxic growth on glucose and lactose



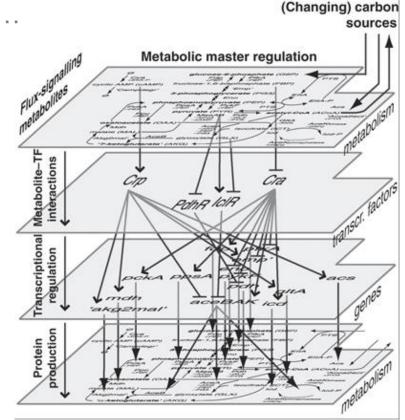
Bettenbrock et al. (2006), J. Biol. Chem., 281(5):2578-84

Adaptation of bacterial physiology to different carbon sources



## **Coordination of adaptative responses**

- Coordination of adaptative responses of bacterial cell achieved by large and complex regulatory networks
  - Variety of molecular mechanisms...
  - ... operating on different timescales...



Kotte et al. (2010), Mol. Syst. Biol., 6: 355



# No global view on network functioning

- Coordination of adaptative responses of bacterial cell achieved by large and complex regulatory networks
- Abundant knowledge on biochemical mechanisms underlying interactions between network components
- Accumulation of data on multi-level response of network to external perturbations

Metabolic fluxes and cellular concentrations of metabolites, enzymes, transcription factors, signalling molecules, ...

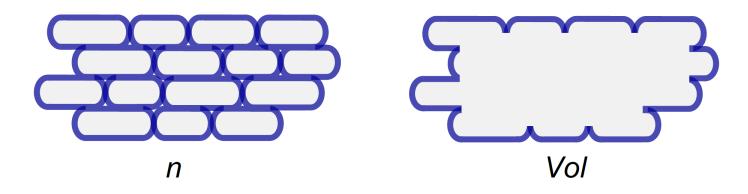
- However, **global view on functioning of entire network** is difficult to achieve and largely absent today
- Use of models to analyse and predict dynamical behaviour of system

Emergence of new discipline: systems biology



 Growth can be considered on the level of number of individual cells or aggregated volume of growing population Vol [L]

Segregated vs nonsegregated models



de Jong et al. (2017), J. Roy. Soc. Interface, 14(136):20170502



• Ordinary differential equation (ODE) model of the growth of a population of microorganisms

Growth rate  $\mu$  [h<sup>-1</sup>]

$$\dot{V}ol = \mu \cdot Vol$$

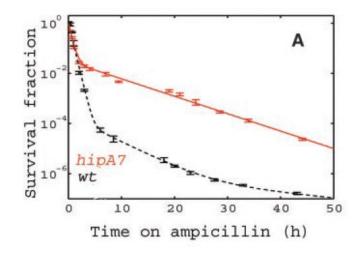
• Solution of growth model for constant growth rate  $\mu = \mu^*$ 

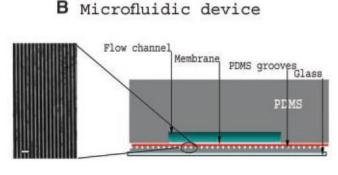
$$Vol(t) = Vol(0) \cdot e^{\mu^* \cdot t}$$

Doubling time  $t_{1/2} = \ln 2/\mu^*$ 



- If all cells have same growth rate, segregated and nonsegregated models are identical
- But: growth rate of cells in population may be heterogeneous
  - Bacterial persistence after antibiotics treatment

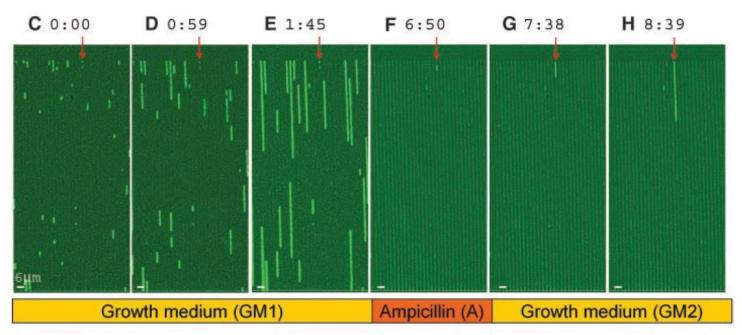




Balaban et al. (2004), Science, 305(5690):1622-5



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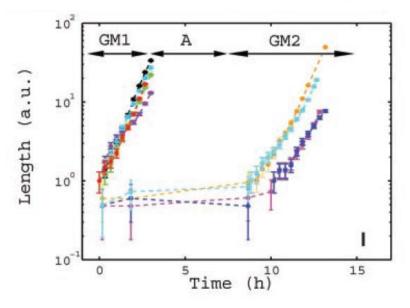


Balaban et al. (2004), Science, 305(5690):1622-5





- If all cells have same growth rate, segregated and nonsegregated models are identical
- But: growth rate of cells in population may be heterogeneous
  - Bacterial persistence after antibiotics treatment
  - Persister cellss have lower growth rate before antibiotics treatment



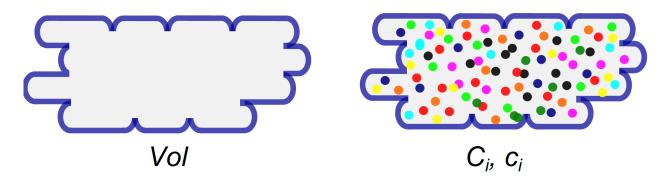
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- Growth is fueled by biochemical processes
- Models describing molecular constituents and biochemical reactions in which they are involved

Structured vs unstructured models





• Basic assumption: volume proportional to biomass (total mass of molecular constituents in cells)

Dry mass of constituent *i* ,  $C_i$  [g] Biomass B [g]

$$Vol \sim \sum_{i} C_i = B$$

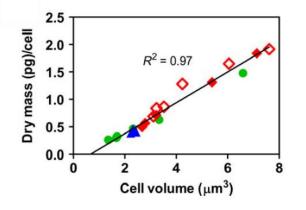
• In other words, biomass density  $1/\delta$  [g L<sup>-1</sup>] is constant:

$$Vol = \delta \cdot \sum_{i} C_i = \delta \cdot B$$



Assumption of constant biomass density supported by experimental data

Biomass density approximately  $300 \text{ g L}^{-1}$ 



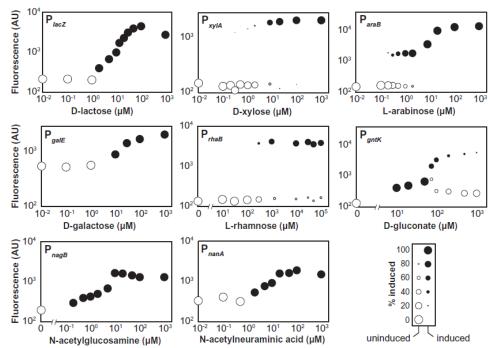
Conditions	Strain	Description	Medium	Symbols
Nutrient limitation	NCM3722	Wild type	Various nutrient	
Translation Inhibition with Cm	NCM3722	Wild type	Glucose with Cm	<b>A</b>
Glucose LacZ OE	NQ1389	Titratable LacZ expression	Glucose with cTc	٠
Glucose +cAA LacZ OE	NQ1389	Titratable LacZ expression	Glucose+cAA with cTc	٥





- Concentration  $c_i \; [g L^{-1}]$  of molecular constituent *i* in population:  $c_i = C_i / Vol$
- If all cells have same concentration, then  $C_i$  also applies to individual cells
- But: concentrations may be heterogeneous over population, leading to different growth phenotypes

Enzymes for secondary carbon sources in *E. coli* 



Afroz et al. (2014), Mol. Microbiol., 93(6):1093-1103



• Concentration  $C_i$  [g L<sup>-1</sup>] of molecular constituent *i* in population:

$$c_i = C_i / Vol$$

- If all cells have same concentration, then  $C_i$  also applies to individual cells
- Consequence of proportionality of mass and volume: total biomass concentration is constant

$$\sum_{i} c_{i} = \sum_{i} C_{i} / Vol = B / Vol = 1/\delta$$



- ODE model of dynamics of molecular constituent *i*
- **Exercise**: write down expression for  $dc/dt = \dot{c}_i$  using its definition  $c_i = C_i/Vol$



• ODE model of dynamics of molecular constituent *i* :

$$\dot{c}_i = \frac{\dot{C}_i \cdot Vol - C_i \cdot \dot{V}ol}{Vol^2} = \frac{\dot{C}_i}{Vol} - \frac{C_i}{Vol} \cdot \frac{\dot{V}ol}{Vol}$$
$$= \frac{\dot{C}_i}{Vol} - \mu \cdot c_i.$$

Appearance of term for growth dilution of individual constituents

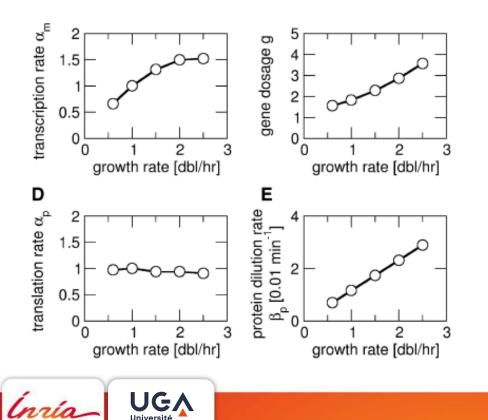
• Growth rate follows from dynamics of molecular constituents

$$\mu = \frac{\dot{V}ol}{Vol} = \delta \cdot \sum_{i} \frac{\dot{C}_{i}}{Vol} = \delta \cdot \frac{\dot{B}}{Vol}$$

No growth dilution if mass of all constituents remains constant



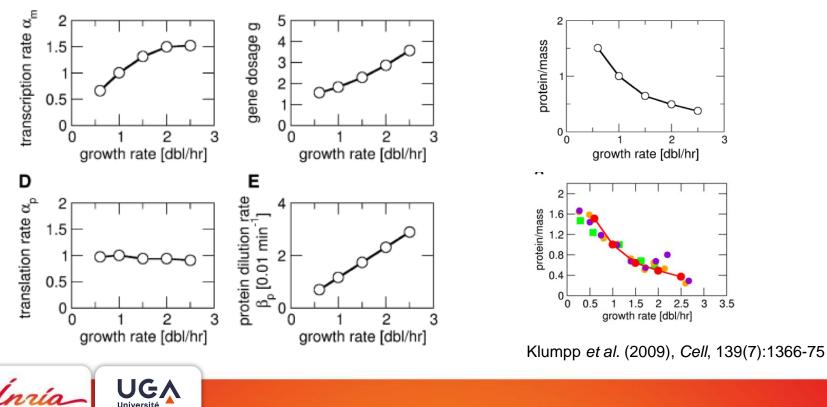
- Growth dilution may have an important effect on the concentration of cellular constituents
  - Changes in rate of protein synthesis and decay of **constitutive gene**



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Klumpp et al. (2009), Cell, 139(7):1366-75

- Growth dilution may have an important effect on the concentration of cellular constituents
  - Changes in rate of protein synthesis and decay of **constitutive gene**
  - Concentration of gene product is growth-rate dependent



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- Term  $\dot{C}_i/Vol$  represents net effect of biochemical reactions on concentration of molecular constituent *i*
- Change of variables using MW:  $X_i = C_i/\alpha_i \text{ [mol]}$ Rate of reactions based on physical encounters of molecules

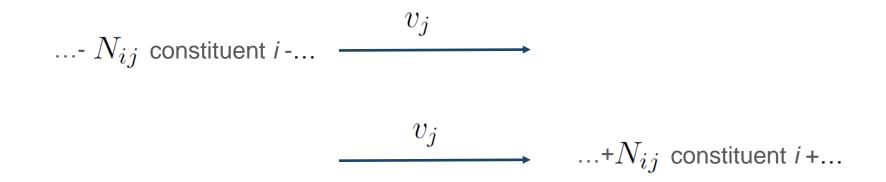
$$x_i = X_i / Vol$$

• ODE model of dynamics of molecular constituent *i* :

$$\dot{x}_i = \frac{\dot{X}_i}{Vol} - \mu \cdot x_i$$



- Reformulation of reaction rates  $\dot{X}_i/Vol$ 
  - Rate of reaction j:  $v_j \pmod{L^{-1} h^{-1}}$
  - Stoichiometry of constituent *i* in reaction *j* :  $N_{ij}$

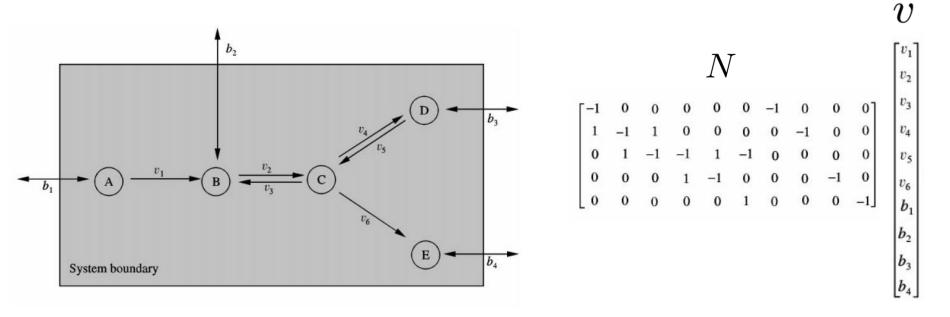


• Example  $2 \text{ A} + 1 \text{ B} \rightarrow 2 \text{ C}$ 



- Stoichiometry matrix  $N\,$  describes structure of reaction network

Internal reactions and exchange reactions, reversible and irreversible



Schilling et al. (2000), J. Theor. Biol., 203(3):229-48



- Reformulation of reaction rates  $\dot{X}_i / Vol$ 
  - Vector of reaction rates: v
  - Stoichiometry of constituent *i* in reaction *j* :  $N_{ij}$
  - Vector of concentrations of molecular constituents: x
- Stoichiometry model of biochemical reactions

$$\dot{x} = N \cdot v - \mu \cdot x$$



• Stoichiometry model of biochemical reactions

$$\dot{x} = N \cdot v - \mu \cdot x$$

• Expression of growth rate

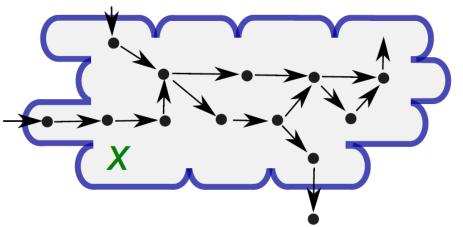
$$\mu = \delta \cdot \sum_{i} \frac{\dot{C}_{i}}{Vol} = \delta \cdot \sum_{i} \alpha_{i} \cdot \frac{\dot{X}_{i}}{Vol}$$
$$= \delta \cdot \sum_{i} \alpha_{i} \cdot N_{i} \cdot v(x).$$

- Rate of accumulation of (mass of) constituents (within unit volume per unit time) relative to total amount of constituents (within unit volume)
- Not ad-hoc definition, but derived from basic assumptions



• ODE model for growth of microbial populations:

$$\dot{x} = N \cdot v(x) - \mu \cdot x,$$
$$\mu = \delta \cdot \sum_{i} \alpha_{i} \cdot N_{i} \cdot v(x).$$



• Reaction rates depend on concentrations *x* of substrates, products, effectors



• Basic (irreversible) enzymatic reaction:

S+E 
$$\xrightarrow{v_1}$$
 C  $\xrightarrow{v_3}$  P+E

- **Exercise**: What is the stoichiometry matrix for this system?
- **Exercise**: What is the corresponding ODE model?



• Basic (irreversible) enzymatic reaction:

S+E 
$$\xrightarrow{v_1}$$
 C  $\xrightarrow{v_3}$  P+E

$$\dot{e}(t) = -v_1 + v_2 + v_3$$
  
 $\dot{s}(t) = -v_1 + v_2$   
 $\dot{c}(t) = v_1 - v_2 - v_3$   
 $\dot{p}(t) = v_3$ 



• Mass-action kinetics is based on fundamental law for rate of biochemical reactions

Rates are proportional to concentrations of reactants

$$\begin{array}{c} \textbf{S+E} \overbrace{v_2}^{v_1} \textbf{C} \xrightarrow{v_3} \textbf{P+E} \\ \dot{e}(t) = -v_1 + v_2 + v_3 = -k_f \cdot e(t) \cdot s(t) + k_r \cdot c(t) + k_{cat} \cdot c(t), \\ \dot{s}(t) = -v_1 + v_2 = -k_f \cdot e(t) \cdot s(t) + k_r \cdot c(t), \\ \dot{c}(t) = v_1 - v_2 - v_3 = k_f \cdot e(t) \cdot s(t) - k_r \cdot c(t) - k_{cat} \cdot c(t), \\ \dot{p}(t) = v_3 = k_{cat} \cdot c(t), \end{array}$$

where the following conservation relations hold:

$$e(t) + c(t) = e_0,$$
  
 $s(t) + c(t) + p(t) = s_0, \quad c(0) = 0, \quad p(0) = 0.$ 



• **Mass-action kinetics** is based on fundamental law for rate of biochemical reactions

Rates are proportional to concentrations of reactants

S+E 
$$\xrightarrow{v_1}$$
 C  $\xrightarrow{v_3}$  P+E

- Exercise: What are the units of the parameters if concentrations are expressed in mol L<sup>-1</sup>?
- Exercise: How can the equation system be simplified using the conservation relations? Hint: keep s and c



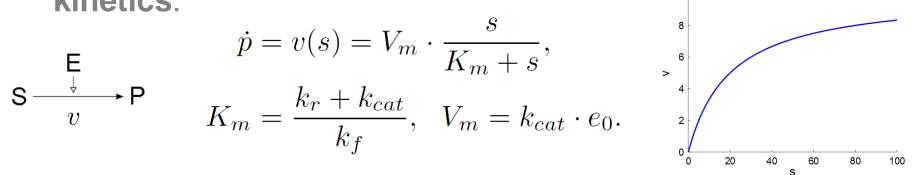
may(Ligh) [[(milth) mol/L 1/h mol/L S=-lef.e.S+ler.c (i) c= hf.e.s - ler.c - heat.c (ii) (iii) e+c=eo  $S+C+p=S_0$ (iv)  $\dot{s} = -k_{f}(e_{o}-c)\cdot s + k_{r}\cdot c$  $\dot{c} = k_{f}(e_{o}-c)\cdot s - (k_{r}+k_{r})\cdot c$ (i) + (iii) +(i) + (iii) :



• Simplified equation system for enzymatic reaction:

$$\dot{s}(t) = -k_f \cdot (e_0 - c(t)) \cdot s(t) + k_r \cdot c(t), \dot{c}(t) = k_f \cdot (e_0 - c(t)) \cdot s(t) - (k_r + k_{cat}) \cdot c(t).$$

- Quasi-steady state assumption:  $\dot{c}(t) \approx 0$
- Quasi-steady state assumption leads to Michaelis-Menten kinetics:



• Exercise: Derive Michaelis-Menten equation



$$\dot{c} = 0 \implies lef \cdot e_0 \cdot s - (ler + lecal + lef \cdot s) c = 0$$
  
$$\implies c = \frac{lef \cdot e_0 \cdot s}{ler + lecal + lef \cdot s} = e_0 \cdot \frac{s}{ler + lecal} + s$$
  
$$= e_0 \frac{s}{K_m + s}, \quad K_m = \frac{ler + lecal}{lef}.$$

$$\dot{p} = lical \cdot c = e_0 \cdot lical \cdot \frac{S}{K_m + S} = V_m \cdot \frac{S}{K_m + S}$$
  
 $V_m \equiv lical \cdot e_0$ 



• Simplified equation system for enzymatic reaction:

$$\dot{s}(t) = -k_f \cdot (e_0 - c(t)) \cdot s(t) + k_r \cdot c(t), \dot{c}(t) = k_f \cdot (e_0 - c(t)) \cdot s(t) - (k_r + k_{cat}) \cdot c(t).$$

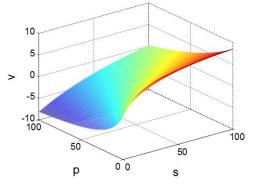
- Quasi-steady state assumption:  $\dot{c}(t) \approx 0$
- Quasi-steady state assumption leads to Michaelis-Menten kinetics
- Quasi-steady state assumption valid under certain conditions on the parameters

Chen et al (2010), Genes Dev., 24(17):1861-75



Michaelis-Menten kinetics for reversible enzymatic reaction

$$\mathbf{S} \longleftrightarrow \mathbf{P} \qquad v(s,p) = \frac{V_m^+ \cdot s/K_{m1} - V_m^- \cdot p/K_{m2}}{1 + s/K_{m1} + p/K_{m2}}$$



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Michaelis-Menten kinetics for reversible enzymatic reaction
 with competitive enzyme inhibition
 No inhibitor

$$E + I \longleftrightarrow E + I \longleftrightarrow E + I \longleftrightarrow E + I \longleftrightarrow P$$

$$v(s, p, i) = \frac{V_m^+ \cdot s/K_{m1} - V_m^- \cdot p/K_{m2}}{1 + i/K_i + s/K_{m1} + p/K_{m2}} \xrightarrow{\int_{0}^{5} \frac{1}{100} \int_{0}^{5} \frac{1}{100} \int$$

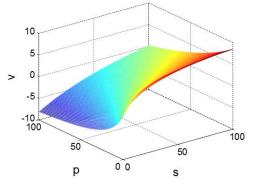


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Michaelis-Menten kinetics for reversible enzymatic reaction

$$\mathbf{S} \longleftrightarrow \mathbf{P} \qquad v(s,p) = \frac{V_m^+ \cdot s/K_{m1} - V_m^- \cdot p/K_{m2}}{1 + s/K_{m1} + p/K_{m2}}$$



Michaelis-Menten kinetics for reversible enzymatic reaction
 with competitive enzyme inhibition

$$E + I \longleftrightarrow E + I \longleftrightarrow E + I \longleftrightarrow E + I \longleftrightarrow V(s, p, i) = \frac{V_m^+ \cdot s/K_{m1} - V_m^- \cdot p/K_{m2}}{1 + i/K_i + s/K_{m1} + p/K_{m2}}$$

#### **Enzyme kinetics**

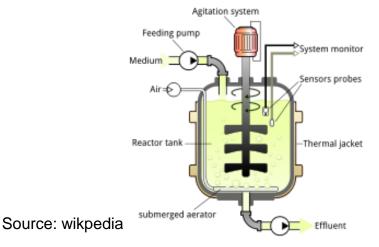
- Many other rate laws for enzyme kinetics have been proposed
  - Generalization to multiple substrates and products
  - Thermodynamic view, separating enzyme-dependent from enzymeindependent properties
  - Convenient mathematical approximations

Heinrich and Schuster (1996), *The Regulation of Cellular Systems*, Chapman & Hall Cornish-Bowden (2004), *Fundamentals of Enzyme Kinetics*, Portland Press

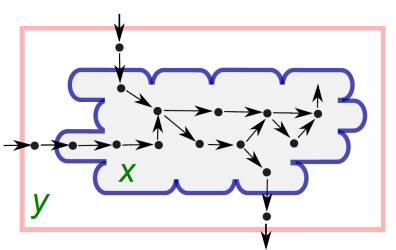
 Rate laws for gene expression kinetics and signal transduction kinetics introduced in later courses



- No explicit model of the environment
  - Some reactions in  $\boldsymbol{v}$  correspond to uptake of substrates or secretion of products
- Environment modeled as bioreactor filled by liquid medium of fixed volume
  - Substrate/product concentrations in medium:  $y [g L^{-1}]$
  - Volume of medium: Vol<sub>medium</sub> [L]



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- Environment modeled as bioreactor filled by liquid medium of fixed volume
  - Substrate/product concentrations in medium:  $y [g L^{-1}]$
  - Volume of medium: Vol<sub>medium</sub> [L]
- ODE model for dynamics of substrate/product concentrations in medium

$$\dot{y} = \alpha_y \cdot E \cdot v(x, y) \cdot (\operatorname{Vol}/\operatorname{Vol}_{medium})$$

- Stoichiometry matrix for exchange reactions:  ${\cal E}$
- Diagonal matrix of molar mass coefficients:  $\alpha_y$





- No explicit model of the environment
  - Some reactions in  $\boldsymbol{v}$  correspond to uptake of substrates or secretion of products
- Environment modeled as bioreactor filled by liquid medium of fixed volume
  - Substrate/product concentrations in medium:  $y [g L^{-1}]$
  - Volume of medium: *Vol<sub>medium</sub>* [L]
- ODE model for dynamics of substrate/product concentrations in medium

$$\frac{Vol}{Vol_{medium}} = \delta \cdot \frac{\sum_{i} C_{i}}{Vol_{medium}} = \delta \cdot b,$$

$$\dot{y} = \delta \cdot \alpha_y \cdot E \cdot v(x, y) \cdot b.$$



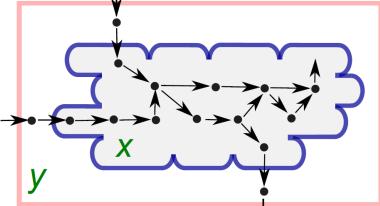
• ODE model for growth of microbial populations:

$$\dot{x} = N \cdot v(x, y) - \mu \cdot x,$$
  

$$\dot{y} = \delta \cdot \alpha_y \cdot E \cdot v(x, y) \cdot b,$$
  

$$\mu = \delta \cdot \sum_i \alpha_i \cdot N_i \cdot v(x, y),$$
  

$$\dot{b} = \mu \cdot b,$$





• ODE model for growth of microbial populations:

$$\begin{split} \dot{x} &= N \cdot v(x, y) - \mu \cdot x, \\ \dot{y} &= \delta \cdot \alpha_y \cdot E \cdot v(x, y) \cdot b, \\ \mu &= \delta \cdot \sum_i \alpha_i \cdot N_i \cdot v(x, y), \\ \dot{b} &= \mu \cdot b, \end{split}$$

 Model applies to batch cultivation, but can be easily adapted for continuous culture or fed-batch culture

Bastin and Dochin (1990), On-Line Estimation and Adaptive Control of Bioreactors, Elsevier, 1990



- Bioreactor models have been mostly used in context of biotechnological applications
- But: they also apply to complex natural environments, such as digestive tracts of vertebrates and insects

Organ shape and location in horse digestive tract	on Example of organ names	Reactor shape	Modelized reactor	Scheme
	Stomach (human) Rumen (cow) Crop (hoazin) Saccular forestomach (kangaroo) Proctodeum P3 (termite)	Open sac-like reactor	Continuously stirred tank reactor (CSTR)	₽
- Crange	Caecum (rabbit)	Closed sac-like reactor	Batch reactor	▲
Cross Contraction	Large intestine (human)	Large tubular reactor	CSTR in series	$\left  \left  \right  \right\rangle$
Source	Small intestine (human) Tubiform forestomach (kangaroo)	Narrow tubular reactor	Plug-flow reactor	>

Godon et al. (2013), BioEnergy Res., 6(3):1063-81



#### Towards integrated models of the cell

 Integrated models of the cell are emerging, but some interesting precursors exist

Coarse-grained model of an E. coli cell

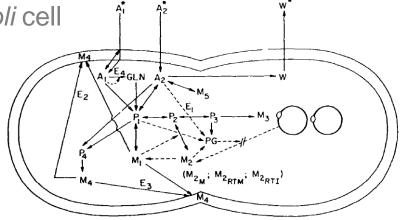


FIGURE 7 An idealized sketch of the model of E. coli B/rA growing in a glucose-ammonium salts medium with glucose or ammonia as the limiting nutrient. At the time shown the cell has just completed a round of DNA replication and initiated cross-wall formation and a new round of DNA replication. Solid lines indicate the flow of material, while dashed lines indicate flow of information. Reproduced with permission from Shuler and Domach, 1983.

- A<sub>1</sub> = ammonium ion
- A<sub>2</sub> = glucose (and associated compounds in the cell)
- W = waste products (CO<sub>2</sub>, H<sub>2</sub>O, and acetate) formed from energy metabolism during aerobic growth
- $P_1$  = amino acids
- $P_2 = ribonucleotides$
- P<sub>3</sub> = deoxyribonucleotides
- $P_4$  = cell envelope precursors
- M<sub>1</sub> = protein (both cytoplasmic and envelope)
- M22m = immature "stable" RNA
- M<sub>2nm</sub> = mature "stable" RNA (r-RNA and r-RNA-assume 85% r-RNA throughout)

- $M_{2_{M}} = messenger RNA$
- $M_3 = DNA$ 
  - $M_4$  = non-protein part of cell envelope (assume 16.7% peptidoglycan, 47.6% lipid, and 35.7% polysaccharide)
  - M, = glycogen
  - PG = ppGpp
- $E_2, E_3$  = molecules involved in directing crosswall formation and cell envelope synthesis—the approach used in the prototype model was used here but more recent experimental support is available
- GLN = glutamine
- $E_1 =$ glutamine synthetase
- \*-the material is present in the external environment

Domach et al. (1984), Biotechnol. Bioeng., 26(3):203-16





#### Towards integrated models of the cell

Integrated models of the cell are emerging, but some interesting precursors exist

Coarse-grained model of an E. coli cell

 Model has evolved into minimal, functionally complete model of chemoheterotrophic bacterium

Model structure	Count	Examples		
Compartments 4		Cytoplasm, cell membrane, whole cell, medium		
Chemical species	408	Glucose-6P, alanine, mRNAs, proteins		
Reactions 570		Fructose-6P synthesis, CTP synthesis		
Rate parameters	570	Mass action or Michaelis-Menten rate constants		
Saturation parameters	581	Michaelis-Menten-like saturation parameters Michaelis-Menten-like inhibition parameters		
Inhibition parameters	25			
Rate rules	1 `	Methylation state of chromosome		
Algebraic rules	1	Cell width (CW)		
Events	36	DNA replication initiation, cell division		
Constraints	408 Each species must have mass >0			
Gencs	,241	Protein and stable RNA coding genes		
Single coding genes	(102	dnaB, pgi, etc.		
Gene clusters				
Genes in clusters 139		Ribosomal proteins, dnaE, etc.		

Shuler et al. (2012), Methods Mol. Biol., 881:573-610

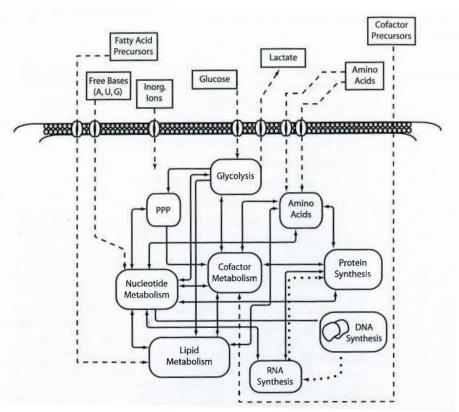


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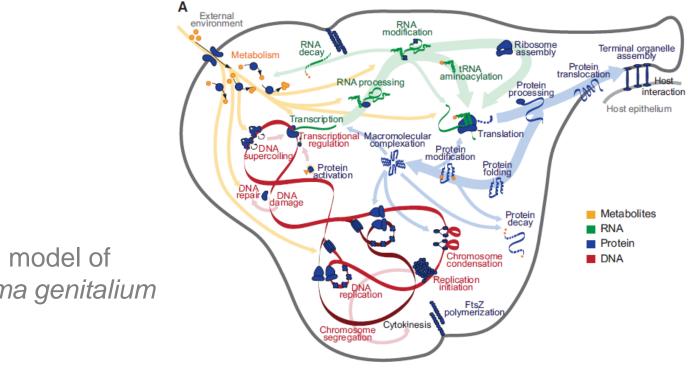
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Metabolic networks are integrated with gene networks and signalling networks

Complex multi-level system with feedback across different timescales

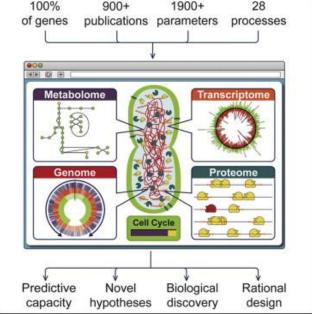


Whole-cell model of Mycoplasma genitalium

Karr et al. (2012), Cell, 150(2): 389-401



- Whole-cell model represents huge modelling effort:
  - Whole-genome model including complete known metabolic, gene, and signalling networks



Karr et al. (2012), Cell, 150(2): 389-401

- Variety of formalisms to model the 28 modules: FBA, kinetic ODE models, Boolean models, Markov chains, …
- Cell cycle simulated for >100 cells, >30 mutants on 128-core machine



• Whole-cell simulation of *M. genitalium* cell cycle

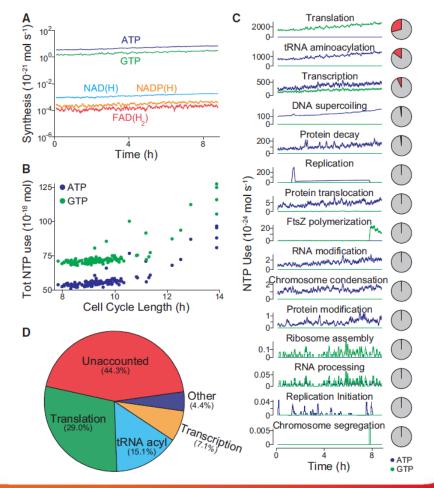


Ínría



- Whole-cell simulations have provided new insights into global use and allocation of energy
- Transcription and translation most costly processes
- Energy use largely independent of cell-cycle length
- Usage of almost half of produced energy not accounted for!

Karr et al. (2012), Cell, 150(2): 389-401





#### Whole-cell models

• Whole-cell models help analyze the dynamics of interactions between multiple functions of the cell

Models allow predictions to be confronted with experimental data and performance of thought experiments

- But whole-cell models have problems as well!
  - Models difficult to construct, to debug and to maintain
  - Huge number of parameters, many unknown: parameter estimation is a difficult problem requiring many data of high quality
  - How do we extract fundamental insights on cell functioning from large, mechanistic models?



#### Whole-cell models

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#### On Exactitude in Science

Jorge Luis Borges, Collected Fictions, translated by Andrew Hurley.

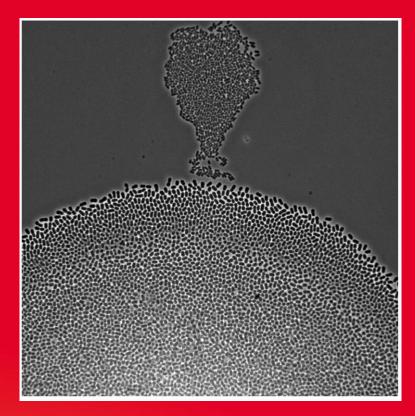
... In that Empire, the Art of Cartography attained such Perfection that the map of a single Province occupied the entirety of a City, and the map of the Empire, the entirety of a Province. In time, those Unconscionable Maps no longer satisfied, and the Cartographers Guilds struck a Map of the Empire whose size was that of the Empire, and which coincided point for point with it. The following Generations, who were not so fond of the Study of Cartography as their Forebears had been, saw that that vast Map was Useless, and not without some Pitilessness was it, that they delivered it up to the Inclemencies of Sun and Winters. In the Deserts of the West, still today, there are Tattered Ruins of that Map, inhabited by Animals and Beggars; in all the Land there is no other Relic of the Disciplines of Geography.

—Suarez Miranda, *Viajes de varones prudentes*, Libro IV, Cap. XLV, Lerida, 1658





# **Thanks!**



#### team.inria.fr/microcosme



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